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Analytical Solution for Nonlinear Convective–radiative Heat Transfer in a Straight Fin using Series Expansion Integration Method

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ABSTRACT
In this work, analytical solutions of nonlinear heat transfer equation in a convective–radiative longitudinal rectangular fin is developed using series expansion integration method. The results of the solution of the series expansion integration method are good agreement with the numerical solution. With the aid of the analytical solution, the effects of convective and radiative heat transfer parameters on the temperature distribution, rate of heat transfer, and thermal efficiency of the longitudinal rectangular fin are investigated. From the results, it is established that the thermal performance of the fin is greatly enhanced under the combined modes of convective and radiative heat transfers. The results obtained in this study serve as the basis for determining the level of accuracy of any other approximation method used in the analysis of the problem. Also, it could be used to improve the design of convecting–radiating fin in heat transfer equipment.

Keywords: *analytical solution; convecting–radiating fins; heat transfer; series expansion integration method.*

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INTRODUCTION

In the design and construction of various types of heat transfer equipment and components, an array of rectangular fins is widely used to enhance heat dissipation from a hot primary surface. The arrangement is mostly effective in a natural convection environment where the convection heat transfer coefficient is low. In this circumstance, the radiative component of heat loss from the fins is comparable to the natural convection heat loss. Therefore, the fin heat transfer model must simultaneously include surface convection and radiation [1]. The enormous applications of this type of fin have aroused interest of various workers as there have been extensive works on the heat transfer characteristics, which attempt to improve the design and provide the optimized mass of the fins. In the attempts of studying the functions, investigating the heat transfer characteristics and optimizing the performance of the fin, the fin equation has been solved with the aid of numerical methods or approximate analytical methods as presented in literature, since an analytical solution is impossible because of the presence of nonlinear terms in the governing differential equations [2]. Aziz and Benzies [3] presented a double series in two perturbation parameters to obtain solution to a convecting–radiating fin with temperature-dependent thermal conductivity. Nguyen and Aziz [4] analyzed the heat transfer from convecting–radiating fins of different profile shapes using numerical integration techniques. Arslanturk [5] used Adomian decomposition method (ADM) to evaluate the fin efficiency of a conductive–convective straight fin with variable thermal conductivity, while Hatami et al. [6] applied least square method (LSM) and fourth-order Runge–Kutta method for the analysis of heat transfer and temperature distribution in circular convective–radiative porous fin with different geometries. Adomian decomposition method and genetic algorithm were used by Singla and Das [7] to predict the heat generation number and fin tip temperature, while Heidarzadeh et al. [8] analyzed the temperature distribution of a convective–radiative fin using ADM. The solution for temperature distribution of convective–radiative fin with nonlinear boundary condition was presented by Chiu and Chen [9] using ADM. Roy et al. [10] adopted ADM for finding the effects of environmental temperature and heat generation on the temperature distribution and efficiency of a convective–radiative fin rectangular straight fin with variable thermal conductivity. Ganji et al. [11] used Galerkin method to solve for the efficiency and temperature distribution of conductive, convective and radiative straight fins. Aziz and Khani [12] studied convection–radiation of a continuously moving fin of variable thermal conductivity. They solved this problem using homotopy analysis method (HAM) and analyzed the effect of several parameters. Miansari et al. [13] applied He's variational iterative method (VIM) to analyze temperature distribution of a convective–radiative fin and other nonlinear equations arising in heat transfer. Differential transform method was used in solution and the effects of several parameters were also studied. Aziz and Torabi [14, 15] studied convective–radiative fin with temperature-dependent thermal properties and surface emissivity, and also of various profile temperatures. Shukla [16] studied the temperature distribution in a sublimation-cooled coated cylinder in convective and radiative environments. Yu and Chen [17] gave rigorous formulations using a Taylor transformation, and investigated the optimal fin length of the convective–radiative rectangular straight fin with variable thermal conductivity. Asadi and Khoshkho [18] presented an exact solution for the temperature distribution in a constant cross-sectional area convecting–radiating fin using integration techniques. Many researchers have studied heat transfer of moving convective–radiative fin. Aziz and Khani [19] proposed to the HAM with 20 terms of series to solve heat transfer in the moving fin with temperature-dependent thermal conductivity and heat losses by both convection and radiation. Aziz and Lopez [20] used a numerical algorithm built into Maple 14 to investigate the thermal processing in a continuously moving rod with variable thermal conductivity and considering both convective and radiative heat losses. Torabi et al. [21] developed the DTM to solve this kind of problem, while Kanth and Kumar [22, 23] adopted the Haar wavelet method (HWM). Recently, Saedodin and Barforoush [24] applied DTM to analyze the thermal processing of moving convective–radiative plates with temperature-dependent thermal conductivity, heat transfer coefficient and surface emissivity. Torabi et al. [25] utilized DTM to develop analytical solution for convective-radiative continuously moving fin with temperature dependent thermal conductivity. Kanth and Kumar [26,27] adopted Haar Wavelet Method to analyze a continuously moving convective-radiative fin with variable thermal conductivity. Saedodin and Barforoush [28] presented a comprehensive analytical study for convective-radiative continuously moving plates with multiple non-linearities.

The heat transfer in convecting–radiating fin occurs nonlinearly and as such it is difficult to find the exact analytical solutions for such problem. Therefore, from the reviewed literatures, numerical methods or approximate analytical methods were applied to solve the problem. However, the classical way for finding analytical solution is obviously still important since it serves as accurate benchmark for numerical solutions. The experimental data are useful to access the mathematical models, but are never sufficient to verify the numerical solutions of the established mathematical models. Comparison between the numerical calculations and experimental data fails to reveal the compensation of modeling deficiencies through the computational errors or unconscious approximations in establishing applicable numerical schemes. Additionally, analytical solutions for specified problems are also essential for the development of efficient applied numerical simulation tools. Also, in practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers. Inevitably, analytical expressions are required to determine the fin temperature distribution, efficiency, effectiveness and the optimum parameter. Analytical solutions, when available, are advantageous in that they provide a good insight into the significance of various system parameters affecting the transport phenomena. Also, analytical expression is more convenient for engineering calculation compared with experimental or numerical studies, and it is obvious starting point for a better understanding of the relationship between physical quantities/properties. Analytical solution provides continuous physical insights than pure numerical/computation method. It helps to reduce the computation cost and task in the analysis of such problem. Also, it is convenient for parametric studies and accounting for the physics of the problem. It appears more appealing than the numerical solution. Therefore, in this work, a series expansion integration method applied to provide analytical solution for the nonlinear differential equations without linearization, discretization or perturbation. The method shows to be advantageous over pure numerical schemes since no discretization is needed and over the known approximate analytical methods for solving nonlinear differential equation since it requires no approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives, and computation of derivatives symbolically. It greatly reduced the size of computational work while still accurately providing the series solution with fast convergence rate. Unlike most numerical techniques, it provides a closed-form solution. It provides good results to the solution of nonlinear equation with high accuracy. The method does not require many computations as carried out in DTM, HPM, HAM, ADM and VIM to have high and fast rate of convergence.

FORMULATION OF THE PROBLEM

Consider a straight fin of rectangular profile (Figure 1) with a cross-sectional area A, length L, constant thermal conductivity k, and surface emissivity ε, exposed to an environment. The fin is attached to a primary surface at fixed and uniform temperature T_b. The primary surface temperature is greater than the ambient temperature, so that the heat is dissipated from the fin surface by simultaneous convection and radiation through its surfaces to the surrounding medium. The sink (the medium surrounding the fin) temperatures for convection and radiation are uniform and they are T_c and T_r, respectively. Assuming the heat flow in the fin and its temperatures remain constant with time, the convective heat transfer coefficient (h) on the faces of the fin is constant and uniform over the entire surface of the fin, the fin thickness is small, compared with its height and length, so that temperature gradients across the fin thickness and heat transfer from the edges of the fin may be neglected, no contact resistance where the base of the fin joins the prime surface, no heat sources within the fin itself and the heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium, the thermal energy balance on volume element can be expressed as follows:

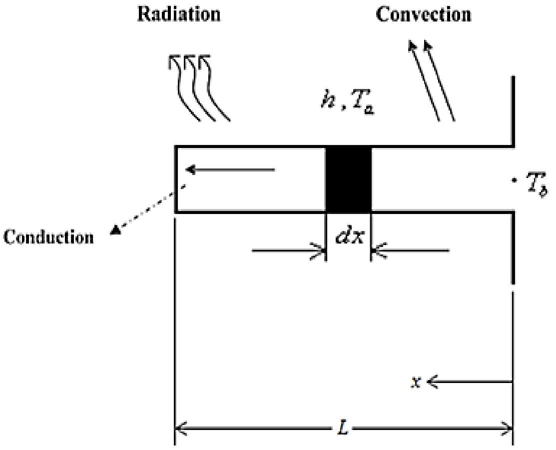

Fig. 1. *The geometry of straight rectangular convective–radiative fin.*

Rate of heat conduction into the element at x = Rate of heat conduction from the element at $x+dx$

+ Rate of heat convection from the element (1)

+ Rate of heat radiation from the element

Mathematically, the thermal energy balance could be expressed as follows:

$$Q_x = Q_{x+dx} + Q_{conv.} + Q_{rad.} \quad (2)$$

i.e.,

$$Q_x - Q_{x+dx} = Q_{conv.} + Q_{rad.} \quad (3)$$

$$Q_x - \left(Q_x + \frac{\delta q}{\delta x}dx\right) = hP(T - T_c)dx + \sigma\epsilon P(T^4 - T_r^4)dx$$

As $dx \rightarrow 0$, Equation (3) reduces to Equation (4):

$$-\frac{dq}{dx} = hP(T - T_c) + \sigma \in P\left(T^4 - T_r^4\right) \quad (4)$$

From Fourier's law of heat conduction,

$$q = -kA_{cr}\frac{dT}{dx} \quad (5)$$

Substituting Equation (5) into Equation (4), the following equation was obtained:

$$\frac{d}{dx}\left(kA_{cr}\frac{dT}{dx}\right) = hP(T - T_c) + \sigma \in P\left(T^4 - T_r^4\right) \quad (6)$$

Following the assumptions that the cross-sectional area, A_{cr}, and the constant thermal conductivity k, are constant, Equation (6) results into

$$kA_{cr}\frac{d^2T}{dx^2} = hP(T - T_c) + \sigma \in P\left(T^4 - T_r^4\right) \quad (7)$$

Dividing Equation (7) through by kA_{cr}, then the governing differential equation for the convecting–radiating fin, as given by Equation (8), arrived at

$$\frac{d^2T}{dx^2} - \frac{hP(T - T_c)}{kA_{cr}} - \frac{\sigma \in P}{kA_{cr}}\left(T^4 - T_r^4\right) = 0 \quad (8)$$

while the boundary conditions are

$$x = 0, \quad T = T_b, \quad (9)$$

$$x = L, \quad -k\frac{\partial T}{\partial x} = h(T - T_c),$$

If negligible heat loss at the tip of the fin is considered, then boundary conditions are

$$x = 0, \quad T = T_b, \quad (10)$$

$$x = L, \quad \frac{\partial T}{\partial x} = 0$$

Applying the following dimensionless parameters,

$$X=\frac{x}{L},\quad \theta=\frac{T}{T_b},\quad \theta_c=\frac{T_c}{T_b},\quad \theta=\frac{T_r}{T_b},\quad M_c^2=\frac{hPL^2}{kA_{cr}},\quad M_r^2=\frac{\sigma\in PL^2T_b^3}{kA_{cr}} \tag{11}$$

Equations (12) and (13) were obtained:

$$\frac{d^2\theta}{dX^2}-M_c^2(\theta-\theta_c)-M_r^2\left(\theta^4-\theta_r^4\right)=0 \tag{12}$$

The boundary conditions are

$$X=0,\quad \theta=1$$
$$X=1,\quad \frac{\partial\theta}{\partial X}=0 \tag{13}$$

It is very important to point out that the thermo-geometric parameter or the fin convective performance factor, M_c, could be written in terms of Biot number, Bi, and the aspect ratio, a_r, as follows:

$$M_c^2=\frac{Ph_bL^2}{A_ck_a}=\frac{(2L)h_bL^2}{(L\delta)k_a}=\frac{2h_b\delta L^2}{\delta^2k_a}=\frac{2h_b\delta}{k_a}\left(\frac{L}{\delta}\right)^2=2Bia_r^2 \tag{14}$$

$$where\quad Bi=\frac{h_b\delta}{k_a},\ a_r=\frac{L}{\delta}$$

From Equation (14), it implies that $M_c^2=a_r\sqrt{2Bi}$

Where Bi is the Biot number = (heat transfer by convection)/ (heat transfer by conduction)

$\gamma=\frac{M_c^2}{M_r^2}=\frac{h}{\sigma\varepsilon T^3}$ = Stanton number/Thring number = (heat transfer by convection)/ (heat transfer by radiation)

Ω= Bi / γ= (heat transfer by radiation) / (heat transfer by conduction)

ANALYTICAL SOLUTION BY SERIES EXPANSION INTEGRATION METHOD

On multiplying Equation (8) through by $\frac{d\theta}{dX}$ and then integrate, Equation (15) is obtained:

$$\frac{1}{2}\left(\frac{d\theta}{dX}\right)^2=M_c^2\left(\frac{\theta^2}{2}-\theta\theta_c\right)+M_r^2\left(\frac{\theta^5}{5}-\theta\theta_r^4\right)+C_1 \tag{15}$$

Applying the fin tip boundary condition in Equation (10), i.e.,

$$X=1,\quad \frac{d\theta}{dX}=0\quad\Rightarrow\theta=\theta_t$$

Equation (15) gives Equation (16):

$$\frac{1}{2}\left(\frac{d\theta}{dX}\right)^2 = M_c^2\left[\frac{1}{2}\left(\theta^2-\theta_t^2\right)-\theta_c(\theta-\theta_t)\right]+M_r^2\left[\frac{1}{5}\left(\theta^5-\theta_t^5\right)-\theta_r^4(\theta-\theta_t)\right] \tag{16}$$

From Equation (16), it can be easily shown that

$$\sqrt{2}X = -\int\left[\frac{M_r^2}{5}\theta^5+\frac{M_c^2}{2}\theta^2-\left(M_c^2\theta_c+M_r^2\theta_r^4\right)\theta-\left(\frac{M_c^2\theta_t^2}{2}-M_c^2\theta_t\theta_c+\frac{M_r^2}{5}\theta_t^5-M_r^2\theta_r^4\theta_t\right)\right]^{-0.5}d\theta \tag{17}$$

Since θ decreases as x increases, the minus sign is used when taking the square root.

Integrating Equation (17) by series expansion up to the sixth term gives

$$\sqrt{2}X = -\int\left[A\theta^5+B\theta^2+C\theta+D\right]^{-0.5}d\theta = \frac{-1}{D^{\frac{1}{2}}}\left[\begin{array}{l}\frac{1}{6}\left(\frac{35BC^3}{32D^4}-\frac{15B^2C}{16D^3}-\frac{63C^5}{256D^5}-\frac{A}{2D}\right)\theta^6\\+\frac{1}{5}\left(\frac{3B^2}{8D^2}+\frac{35C^4}{128D^4}-\frac{15BC^2}{16D^3}\right)\theta^5\\+\frac{1}{4}\left(\frac{3BC}{4D^2}-\frac{5C^3}{16D^3}\right)\theta^4+\frac{1}{3}\left(\frac{3C^2}{8D^2}-\frac{B}{2D}\right)\theta^3\\-\frac{C}{4D}\theta^2+\theta\end{array}\right]+\psi \tag{18}$$

where

$$A=\frac{M_r^2}{5} \qquad B=\frac{M_c^2}{2} \qquad C=-(M_c^2\theta_c+M_r^2\theta_r^4) \qquad D=M_c^2\theta_c\theta_t-\frac{M_r^2\theta_t^5}{5}+M_r^2\theta_r^4\theta_t-\frac{M_c^2\theta_t^2}{2}$$

And ψ is an arbitrary constant

Thus, the LHS of Equation (18) could be written as

$$\sqrt{2}X = \beta_1\theta+\beta_2\theta^2+\beta_3\theta^3+\beta_4\theta^4+\beta_5\theta^5+\beta_6\theta^6+\psi \tag{19}$$

where

$$\beta_1=\frac{-1}{D^{\frac{1}{2}}} \qquad \beta_2=\frac{C}{4D^{\frac{3}{2}}} \qquad \beta_3=\frac{-1}{3D^{\frac{1}{2}}}\left(\frac{3C^2}{8D^2}-\frac{B}{2D}\right) \qquad \beta_4=\frac{-1}{4D^{\frac{1}{2}}}\left(\frac{3BC}{4D^2}-\frac{5C^3}{16D^3}\right)$$

$$\beta_5=\frac{-1}{5D^{\frac{1}{2}}}\left(\frac{3B^2}{8D^2}+\frac{35C^4}{128D^4}-\frac{15BC^2}{16D^3}\right) \qquad \beta_6=\frac{-1}{6D^{\frac{1}{2}}}\left(\frac{35BC^3}{32D^4}-\frac{15B^2C}{16D^3}-\frac{63C^5}{256D^5}-\frac{A}{2D}\right)$$

Using the first boundary condition in Equation (19), the arbitrary constant gives

$$\psi = -\left(\beta_1+\beta_2+\beta_3+\beta_4+\beta_5+\beta_6\right) \tag{20}$$

On substituting Equation (20) into Equation (19), Equation (21) was arrived at

$$\sqrt{2}X = \beta_1(\theta-1)+\beta_2\left(\theta^2-1\right)+\beta_3\left(\theta^3-1\right)+\beta_4\left(\theta^4-1\right)+\beta_5\left(\theta^5-1\right)+\beta_6\left(\theta^6-1\right) \tag{21}$$

Equation (19) or Equation (21) could be written as follows:

$$\sqrt{2}X-\psi = \beta_1\theta+\beta_2\theta^2+\beta_3\theta^3+\beta_4\theta^4+\beta_5\theta^5+\beta_6\theta^6 \tag{22}$$

where the arbitrary constant ψ is given by Equation (20).

Then, by reversion of power series, Equation (22) can be written as follows:

$$\theta=\lambda_1(\sqrt{2}X-\psi)+\lambda_2(\sqrt{2}X-\psi)^2+\lambda_3(\sqrt{2}X-\psi)^3+\lambda_4(\sqrt{2}X-\psi)^4+\lambda_5(\sqrt{2}X-\psi)^5+\lambda_6(\sqrt{2}X-\psi)^6 \tag{23}$$

where

$$\lambda_1=\frac{1}{\beta_1} \qquad \lambda_2=-\frac{\beta_2}{\beta_1^3} \qquad \lambda_3=\frac{2\beta_2^2-\beta_1\beta_3}{\beta_1^5}$$

$$\lambda_4=\frac{5\beta_1\beta_2\beta_3-5\beta_2^3-\beta_1^2\beta_4}{\beta_1^7} \qquad \lambda_5=\frac{6\beta_1^2\beta_2\beta_4+3\beta_1^2\beta_3^2-\beta_1^3\beta_5+14\beta_2^4-21\beta_1\beta_2^2\beta_3}{\beta_1^9}$$

$$\lambda_6=\frac{7\beta_1^3\beta_2\beta_5+84\beta_1\beta_2^3\beta_3+7\beta_1^3\beta_3\beta_4-28\beta_1^2\beta_2\beta_3^2-\beta_1^4\beta_6-28\beta_1^2c_2^2c_4-42c_2^5}{\beta_1^{11}}$$

ANALYSIS OF FIN PERFORMANCE INDICATORS

In order to evaluate the performance of the fin based on engineering parameters of interests, the total surface heat loss and efficiency of the fin are analyzed.

Total Surface Heat Loss

The instantaneous total surface heat loss is the sum of convective and radiative losses as given by Equation (22):

$$Q_{actual}=\int_0^L\left[hp(T-T_\infty)+\sigma\in P(T^4-T_s^4)\right] \tag{24}$$

The ideal fin heat transfer is the heat transfer from the fin when the entire fin surface was at the base temperature. Thus, the idea heat transfer from the fin is given as

$$Q_{ideal}=hPL(T_b-T_\infty)+\sigma\in PL(T_b^4-T_s^4) \tag{25}$$

The Fin Efficiency

The efficiency of thee fin is given as

$$\eta=\frac{Q_f}{Q_{\max}}=\frac{\int_0^1[Ph(T-T_c)+\sigma\in P(T^4-T_s^4)]dx}{PhL(T_b-T_c)+\sigma\in PL(T_b^4-T_s^4)} \tag{26}$$

The dimensionless for of Equation (26) is given

$$\eta=\frac{\int_0^1[M_c^2(\theta-\theta_c)+M_r^2(\theta^4-\theta_s^4)]dX}{M_c^2(1-\theta_c)+M_r^2(1-\theta_s^4)}=\frac{\int_0^1[\gamma(\theta-\theta_c)+(\theta^4-\theta_s^4)]dX}{\gamma(1-\theta_c)+(1-\theta_s^4)} \tag{27}$$

where

$$\gamma=\frac{M_c^2}{M_r^2}=\frac{h}{\sigma\varepsilon T^3}$$

RESULTS AND DISCUSSION

The results of the developed approximant analytical solutions and the parametric studies are given as Figures 2–8. The influences of convective and radiative heat transfer on the fin are shown in Figure 2. It is depicted in the figure that the thermal performance of the fin is enhanced under the combined modes of convective and radiative heat transfer. However, the radiative heat transfer can be neglected if the base temperature of the fin is low and the emissivity of the fin surface is near zero. The important thing for the considerations of radiative heat transfer in fins surface is the emissive because high emissivity gives a great amount of heat radiation transfer from the fin.

The impacts of convective and radiative heat transfer parameters on the temperature distribution in the fin are shown in Figures 3 and 4, respectively. It is shown that the dimensionless temperature distribution falls monotonically along fin length for all various convective and radiative heat transfer parameters. For larger values of the convective and radiative heat transfer parameters, the large amount of heat is convected or radiated from the fin through its length to environment. In the situation of negligible heat loss from the fin tip (insulated tip) to the environment, the fin temperature decreases along the fin length, the temperature decreasing rate is the same around the fin base area. Also, it is shown that the temperature profiles for the various convective and radiative heat transfer parameters coincide initially at the base of the fin but part away toward the tip of the fin. This is due to the fact that convective and radiative heat transfer parameters are factors/multipliers of the temperature difference between the fin surface and surrounding medium (T–T_∞). Such temperature difference between the fin surface and the surrounding decreases from the fin base to the fin tip irrespective of the increase value of the convective heat transfer parameter.

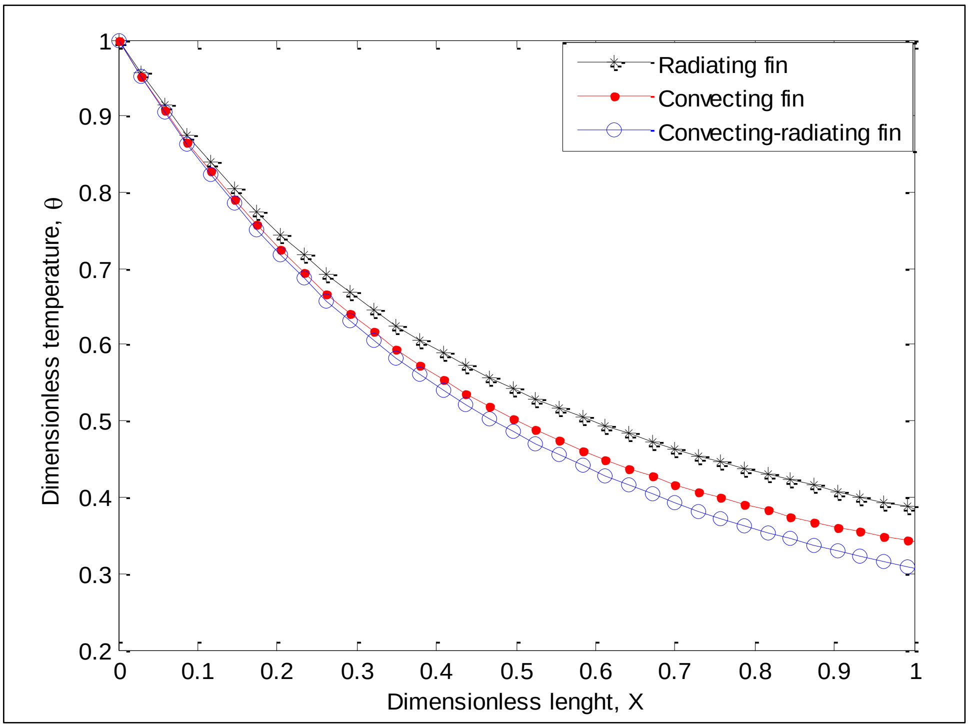

Fig. 2. *Effects of heat transfer mode on the thermal performance of the fin.*

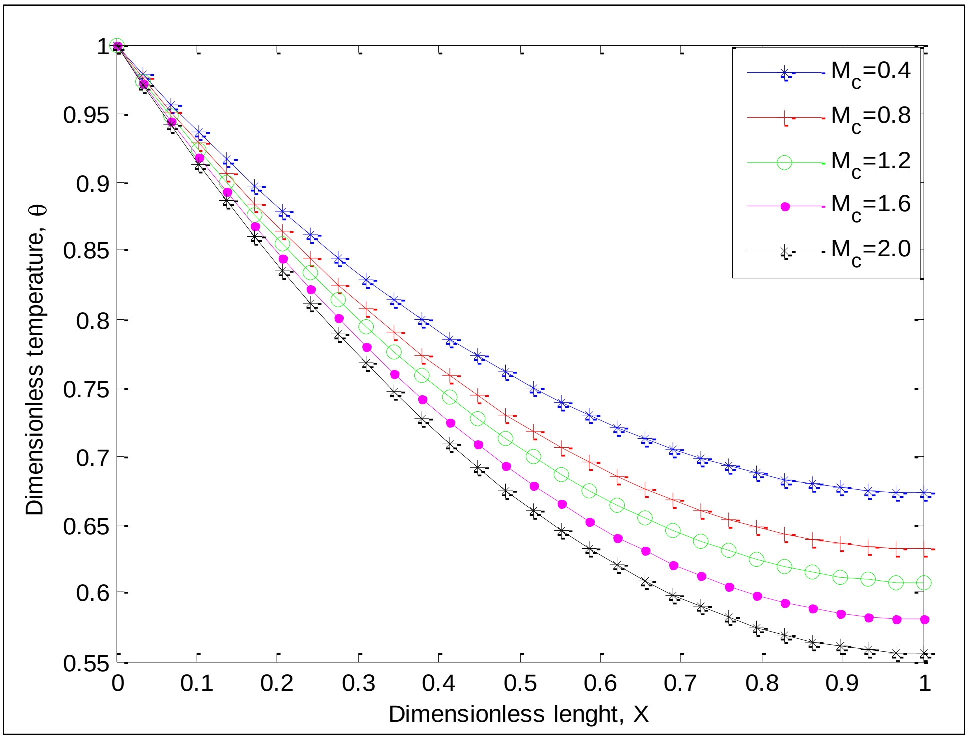

Fig. 3. *Effects of convective parameter on the thermal performance of the fin.*

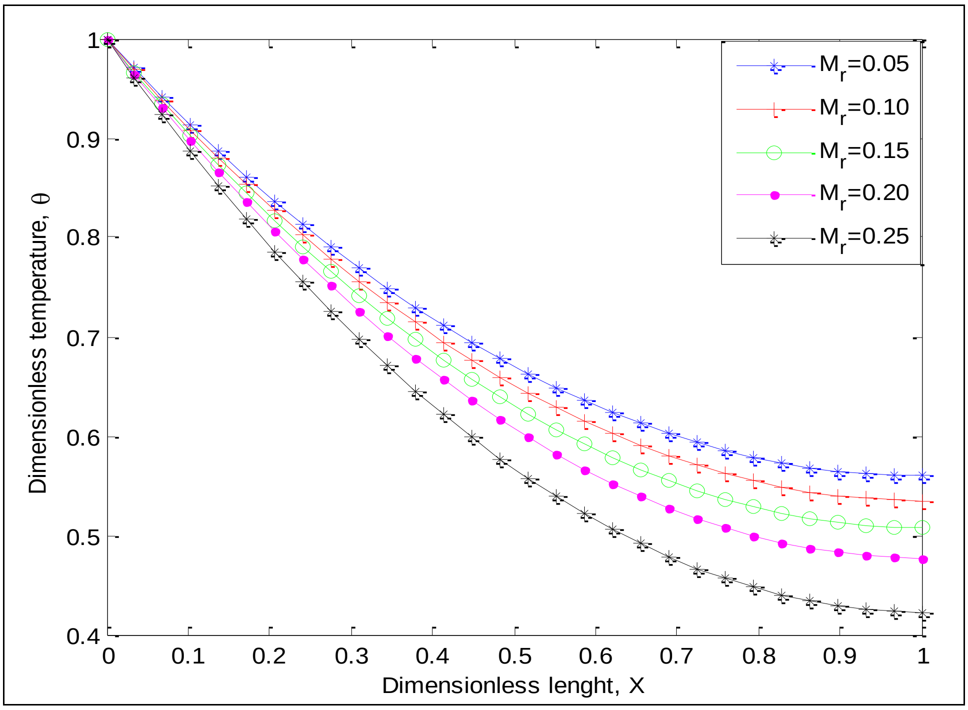

Fig. 4. *Effects of radiative parameter on the thermal performance of the fin.*

Figure 5 shows the effects of convective–radiative parameter ((heat transfer by convection) / (heat transfer by radiation)) on the temperature distribution in the fin. It is shown in the figure that temperature distribution in the fin decreases as the convective–radiative parameter increases.

One of the major important analyses in the fin problem is the determination of the rate of heat transfer at the base of the fin. Figure 6 shows the effects of convective–radiative parameter on the dimensionless heat transfer rate along the fin. Also, the figure depicts the variation of the rate of heat transfer with the fin length. From the figure, it could be deducted that the convective–radiative parameter, which is a function of the thermal conductivity, heat transfer coefficient and emissivity of the fin, has direct and significant effects on the rate of heat transfer along and at the base of the fin.

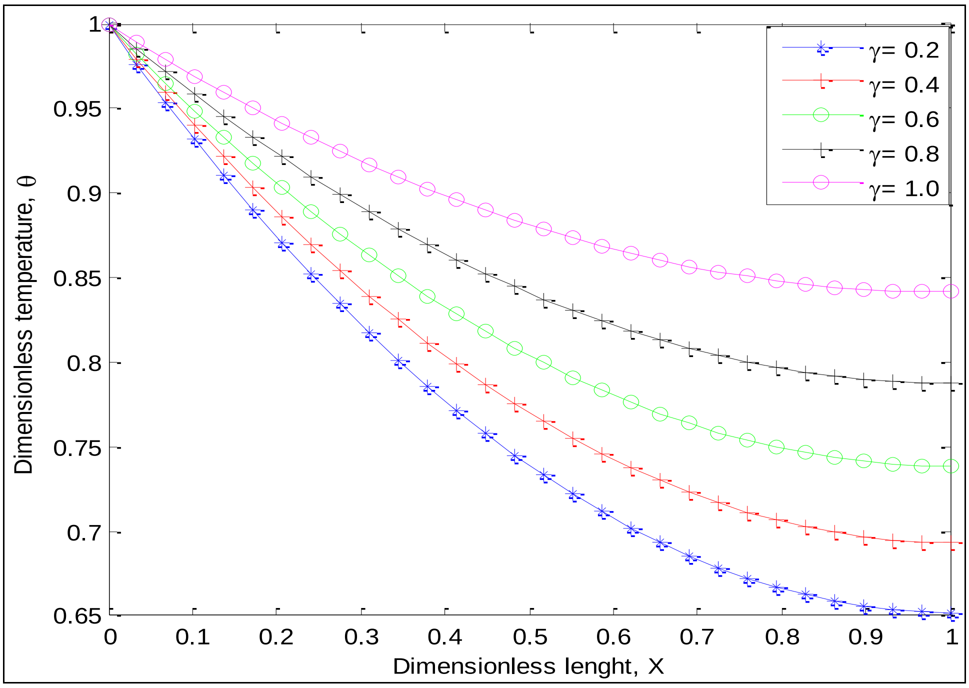

Fig. 5. *Effects of convective-radiative parameter on the thermal performance of the fin.*

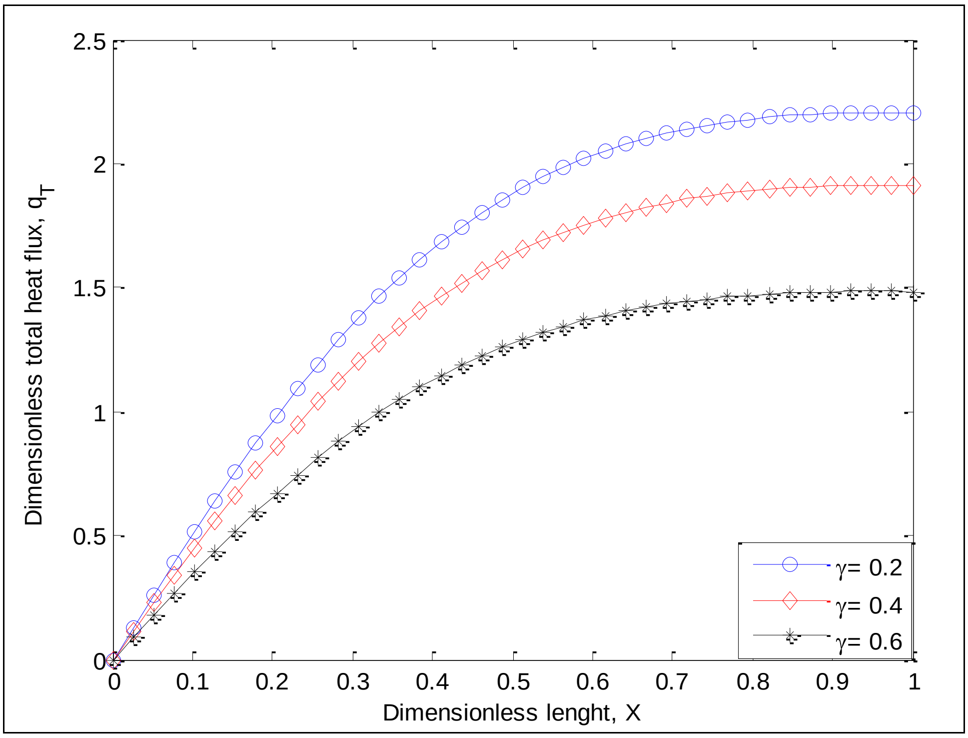

Fig. 6. *Effects of convective–radiative parameter on the heat transfer rate of the fin.*

Figures 7 and 8 show the effects of convective and radiative heat transfer on the efficiency of the fin. It is shown from the results that the fin efficiency decreases monotonically with increasing convective heat transfer parameter. From the figures, it is shown that as the convective heat transfer parameter increases, the efficiency of the fin decreases. When the convective heat transfer parameter equals to zero, the fin efficiency is 100%, which implies that there is no conduction resistance or no presence

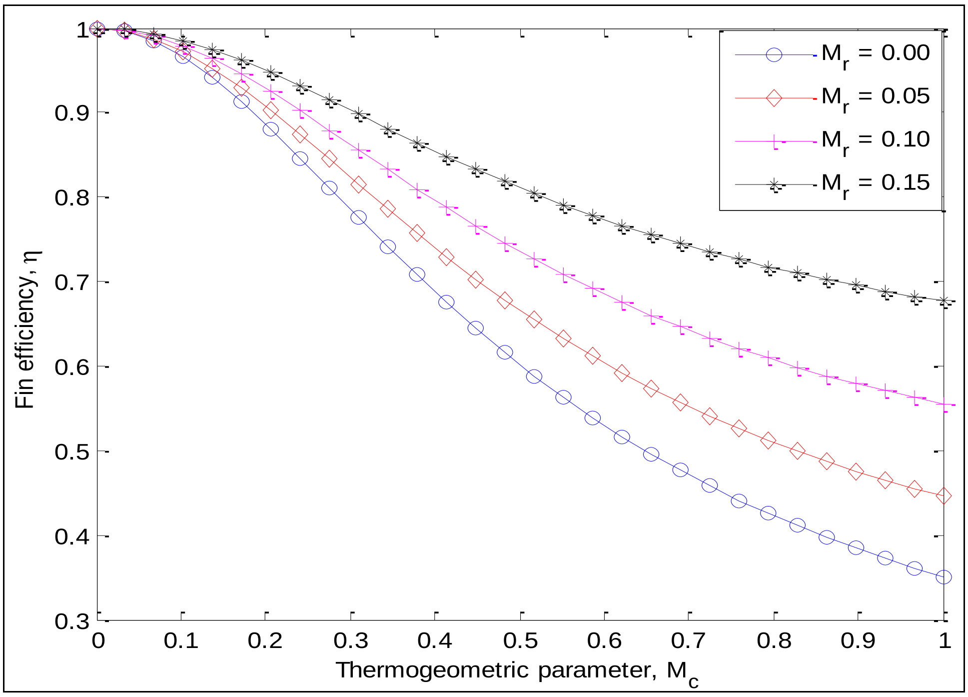

Fig. 7. *Effects of convective–radiative parameter on the thermal efficiency of the fin.*

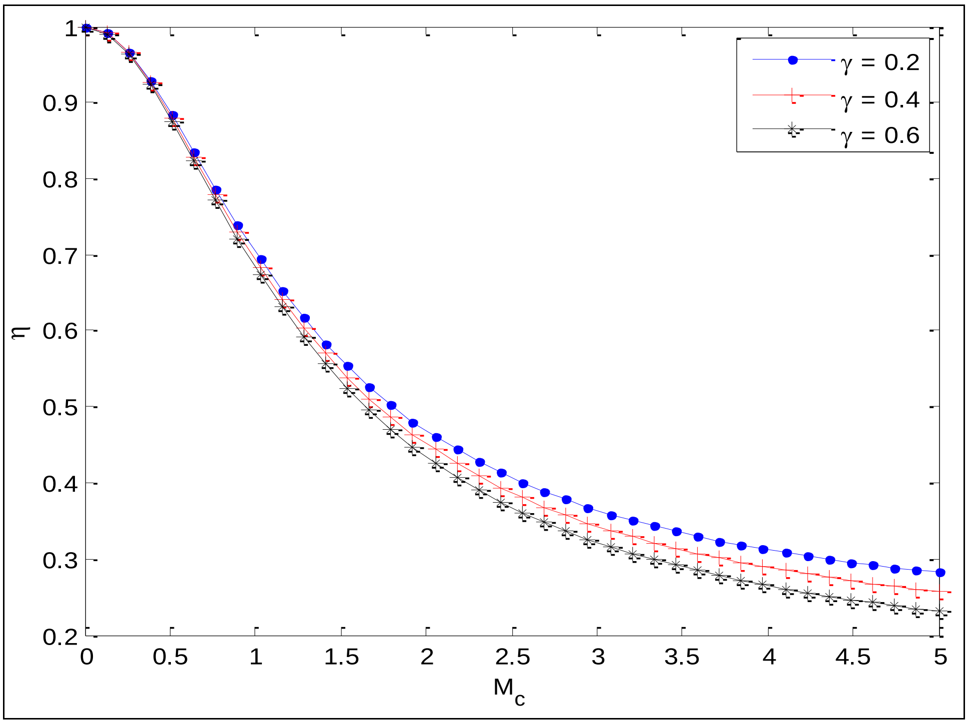

Fig. 8. *Effects of convective–radiative parameter on the thermal efficiency of the fin.*

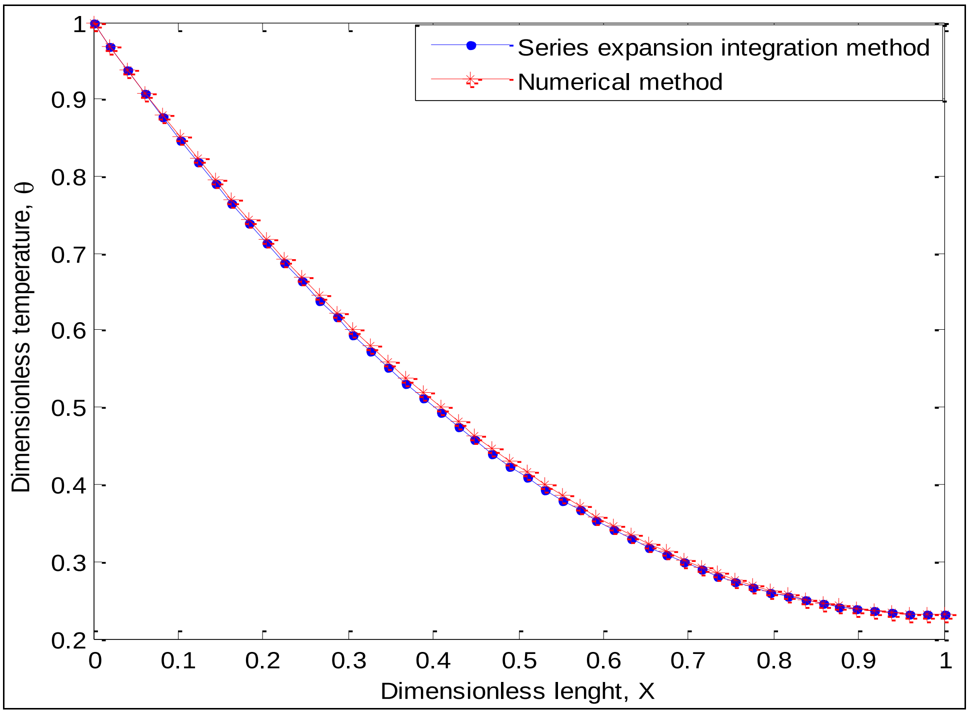

Fig. 9. *Comparison of the series expansion integration method and numerical solutions.*

of fin at all. As the convective heat transfer parameter (ratio of the convective heat transfer coefficient to thermal conductivity) approaches zero, the temperature at every point in the fin is equal to the temperature of the base. The inverse variation in the fin efficiency with the convective heat transfer or thermo-geometric parameter is due to the fact that as more materials are attached to the prime surface, the resistance to heat flow increases thereby reducing the fin efficiency. Upon further increase in the fin thermo-geometric parameter, the effect of reducing the resistance becomes visible in the sense that the fin efficiency starts to normalize. Therefore, high efficiency of the fin could be achieved by using small values of thermo-geometric parameter, which could be realized using a fin of small length or by using a material of better thermal conductivity.

The approximate analytical method of solution is verified with numerical solution using fourth-order Runge–Kutta as shown in Figure 9. From the results, it is depicted that the series expansion integration method displays good agreement with the numerical solution. This fact establishes the high accuracy of the method. Therefore, the application of the method to other nonlinear differential equation is recommended.

NOMENCLATURE

A	cross-sectional area of the fins, m^2
Bi	Biot number
h	heat transfer coefficient, $Wm^{-2}k^{-1}$
k	thermal conductivity of the fin, $Wm^{-1}k^{-1}$
L	length of the fin, m
Mc	dimensionless fin convective heat transfer parameter
Mr	dimensionless fin radiative heat transfer parameter
m^2	fin parameter m^{-1}
P	perimeter of the fin, m
T	temperature, K
x	axial distance, m
X	dimensionless length
Q	dimensionless heat transfer
η	efficiency of the fin

Greek Symbols

δ	thickness of the fin, m
ε	efficiency of the fin
ϵ	emissivity of the fin material
θ	dimensionless temperature

σ Stefan–Boltzmann constant
δ fin thickness

Subscript
b base
c convective
r radiative
t tip

CONCLUSION

In this work, series expansion integration method is used to solve the nonlinear thermal model of convective–radiative cooling fins. It was shown that the series expansion integration method displays good agreement with the numerical solution. From the parametric studies, it was established that the thermal performance of the fin is enhanced under the combined modes of convective and radiative heat transfer. Aside from the fact that the application of the method to other nonlinear differential equation is recommended, the results obtained in this study serve as the basis for determining the level of accuracy of any other approximation methods.

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