

## Thermal Radiation, Melting, and Dispersion Effects on Mixed Convective Heat and Mass Transfer in Fluid Flow Through a Vertical Plate Embedded in a Non-Darcy Porous Medium

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### ABSTRACT

*This paper examines the mixed convective heat transfer effect on a non-Darcy porous fluid flowing through a vertical plate considering concentration and effect of melt on flow and heat transfer. The mathematical models describing the transport phenomena arising from mechanics of the fluid are developed and then analyzed using homotopy perturbation method. The results of the approximate analytical solutions in this present study are verified with the results of the past works as established in literature. Thereafter, the developed approximate analytical solutions are used to investigate the effect of rheological parameters such as melting, thermal radiation and dispersion parameters. From the results, it is established that as the melting parameter increases, the temperature distribution toward center plate increases significantly, while the concentration distribution, it increases toward the upper plate as the melt parameter increases. Also, it was observed that increase in  $R$  parameter causes a corresponding decrease in velocity distribution. Increasing thermal dispersion parameter ( $D$ ) decreases temperature distribution. The present study will be useful as it provides useful insight to applications including fossil fuel combustion, astrophysical flows and geothermal systems amongst others.*

**Keywords:** concentration, heat transfer, melting, mixed convection, non-Darcy medium

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### INTRODUCTION

The influence of melt on heat and mass transfer of fluid flow through porous media has generated wide interests amongst researchers over the years. This is due to its vast practical importance in metal working processes such as magna solidification, welding and casting, heating and cooling chamber, amongst other applications. In efforts to study this exciting flow phenomena, Tien and Yien [1] studied the influence of melt on heat transfer due to convection considering boundary layer theory, while Sparrow et

al. [2] extended the study on mixed convective heat transfer with saturated porous medium analyzing melt in the melt region. In a recent study, Cheng and Lin [3] investigated the influence of melting and external flow on saturated liquid medium during convective mixed heat transfer. Ahmad and Pop [4] reported the mixed convective melt effect of vertical surfaces in saturated porous media considering mixed convection boundary layer flow. Non-Darcy and double dispersion effect on mixed convective heat transfer through vertical surfaces

embedded in porous medium was studied by Afify and Elgazery [5]. A study on chemical reaction effects and double dispersion was presented by El-Amin et al. [6] for non-Darcy convective heat and mass transfer flow. Gorla et al. [7] analyzed mixed convective melt effect in a porous medium, but Prasad and Hemalatha [8] considered mixed convection and thermal dispersion on mixed convective heat transfer in porous medium. Mixed convection effect on heat and mass transfer on non-Newtonian fluid saturated with non-Darcy porous medium was studied by Kairi and Murthy [9]. Murthy [10] analyzed double dispersion effect on mass and heat transfer in non-Darcy porous medium, shortly after Murthy [11] presented the effect of thermal radiation on mixed convective heat and mass transfer on fluid flow through non-Darcy porous medium.

Mathematical models describing mixed convection heat transfer are complex models, which are difficult in obtaining their exact solutions. The simplified form of the complex models deviates the problem from realistic view point. Consequently, recourses are made to numerical methods in order to solve the complex models. However, over the years, various semi- or approximate analytical methods have been developed. These methods include perturbation methods (PM), homotopy analysis method (HAM), variational iteration method (VIM), Adomian decomposition method (ADM), optimal homotopy asymptotic method (OHAM) and differential transformation method (DTM) [12–27]. The PM are limited in applications owing to the problems of weak nonlinearities and artificial perturbation parameter which may be non-existent in practical sense. Also, the need to find an initial condition to satisfy the boundary condition makes methods such as DTM requires

computational tools such as Matlab, Maple or Mathematica in handling a solution of large parameters resulting to large computational cost and time. Lack of rigorous theories for determining the initial approximation of the HAM, its auxiliary function and parameter restricts the HAM. The determination of the Lagrangian parameter or multiplier makes the VIM a tasking approach for less complex. Also, the problem of finding the Adomian polynomials makes the ADM not attractive to researchers but the OHAM requires determining constants using auxiliary functions which may be too rigorous to determine for some nonlinear problems. Sequel from the above, it has been established that among the relatively newly developed approximate analytical methods, homotopy perturbation method (HPM) been shown to be a relatively simplistic method for solving nonlinear equations with high accuracy. Also, it has been found as a favorable analytical technique for solving nonlinear problems. Moreover, a study on simultaneous effects of thermal radiation, melting and viscous dissipation on mixed convective heat and mass transfer in fluid flow through a vertical plate embedded in a non-Darcy porous medium using homotopy perturbation method has not been studied in literature. Therefore, this paper aims to investigate the effect of heat transfer of a non-Darcy flow through vertical plates with concentration and melting effect adopting homotopy perturbation method.

## MODEL DEVELOPMENT AND ANALYTICAL SOLUTION

Consider a steady fluid flow of non-Darcy porous medium that is conveyed through vertical plates considering two-dimensional mixed convective flow as shown in Figure 1. The referenced coordinates are such that  $x$ -coordinate is aligned vertically upwards as the  $y$ -coordinate is aligned normal to the  $x$ -axis.

Constant plate temperature is assumed, constant liquid phase temperature different from the solid phase and interface is assumed. As the surface constitutes solid and liquid phase interface in porous matrix during melt.

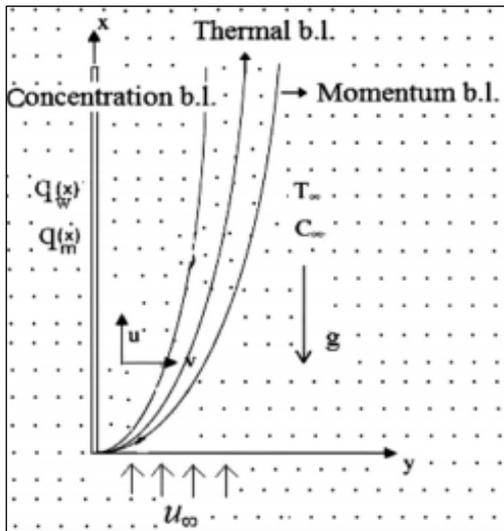


Fig. 1. Schematic diagram and physical coordinate system of the flow process.

With respect to the above assumptions, the boundary layer equations are introduced as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial y} + \frac{C_f \sqrt{K}}{\partial} \frac{\partial}{\partial y} (u^2) \\ = \frac{\rho \infty g k}{v} \left( \beta_T \frac{\partial T}{\partial y} + \beta_C \frac{\partial C}{\partial y} \right) \end{aligned} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \alpha \frac{\partial T}{\partial y} \right) - \frac{1}{\rho \infty C_p} \frac{\partial q_r}{\partial y} \quad (3)$$

$$u \frac{\partial C}{\partial y} + v \frac{\partial C}{\partial y} = \frac{\partial}{\partial y} \left( D_e \frac{\partial C}{\partial y} \right) \quad (4)$$

with boundary layer expressed as

$$\begin{aligned} k \frac{\partial T}{\partial y} = \rho [h_{sf} + c_s (T_m - T_0)] v, T = T_m, \\ C = C_w, y=0 \end{aligned} \quad (5)$$

$$u \rightarrow u_\infty, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y \rightarrow \infty$$

Following the model proposed by Roseland, heat flux due to radiation is expressed as

$$q_r = -\frac{4\sigma^*}{3k} \frac{\partial T^4}{\partial y} \quad (6)$$

where  $T^4$  is expressed linearly with temperature assuming differences in temperature is very small within flow. Expanding  $T^4$  using the Taylor series about  $T_m$ , upon neglecting higher order terms yields  $T^4 \cong 4T_m^3 T - 3T_m^4$ , which is expressed as

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_m^3}{3k^*} \frac{\partial T}{\partial y} \quad (7)$$

Equation (1) is satisfied upon imposing the stream function  $\psi(x, y)$ :

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \quad (8)$$

where  $\psi = \sqrt{\alpha_m u_\infty} x f(\eta)$ ,  $f(\eta)$  is the dimensionless stream function and

$$\eta = \frac{y}{x} \sqrt{\frac{u_\infty x}{\alpha_m}}$$

Velocity components are given by

$$u = u_\infty \frac{df}{d\eta}$$

and

$$v = -\frac{1}{2} \sqrt{\frac{\alpha_m u_\infty}{x}} \left[ f(\eta) - \eta \frac{df}{d\eta}(\eta) \right] \quad (9a)$$

Temperature and concentration are represented as

$$T = T_m + (T_\infty - T_m)\theta(\eta)$$

and

$$C = C_m + (C_\infty - C_m)\phi(\eta) \quad (9b)$$

Therefore, dimensionless equations for flow, temperature and concentration for the two-dimensional boundary value problem are expressed as [28] follows:

$$\left(1 + F \frac{df}{d\eta}\right) \frac{d^2 f}{d\eta^2} + \frac{Ra}{Pe} \left(\frac{d\theta}{d\eta} + N \frac{d\phi}{d\eta}\right) = 0 \tag{10}$$

$$\left(1 + D \frac{df}{d\eta} + \frac{4}{3} R\right) \frac{d^2 \theta}{d\eta^2} + \frac{1}{2} f \frac{d\theta}{d\eta} + D \frac{d^2 f}{d\eta^2} \frac{d\theta}{d\eta} = 0 \tag{11}$$

$$\frac{d^2 \phi}{d\eta^2} + \frac{1}{2} Le f \frac{d\phi}{d\eta} + Le B \left(\frac{df}{d\eta} \frac{d^2 \phi}{d\eta^2} + \frac{d^2 f}{d\eta^2} \frac{d\phi}{d\eta}\right) = 0 \tag{12}$$

with appropriate boundary conditions, given as

$$f(0) + 2M \frac{d\theta}{d\eta}(0) = 0, \frac{df}{d\eta}(\infty) = 1 \tag{13}$$

$$\theta(0) = 0, \theta(\infty) \rightarrow 1 \tag{14}$$

$$\phi(0) = 0, \phi(\infty) \rightarrow 1 \tag{15}$$

where dimensionless parameters are introduced as

$$F = \frac{2C_f \sqrt{Ku_\infty}}{v},$$

$$Ra = \frac{Kg \beta_T \rho_\infty (T_\infty - T_m)}{v \alpha_m},$$

$$Pe = \frac{u_\infty}{\alpha_m},$$

$$N = \frac{C_w - C_\infty}{T_w - T_\infty},$$

$$D = \frac{\gamma du_\infty}{\alpha_m},$$

$$R = \frac{4\sigma^* T_m^3}{kk^*},$$

$$Le = \frac{\alpha_m}{D_1},$$

$$B = \frac{\xi du_\infty}{\alpha_m},$$

$$M = \frac{c_f (T_\infty - T_m)}{h_{sf} + c_s (T_m - T_0)} \tag{16}$$

**Principles of Homotopy Perturbation Method**

The following equation is considered in explaining the fundamentals of the homotopy perturbation method [27]:

$$A(u) - f(r) = 0 \quad r \in \Omega \tag{17}$$

Utilizing the boundary condition,

$$B(u, \frac{\partial u}{\partial \eta}) = 0 \quad r \in \Gamma \tag{18}$$

A is the general differential operator, B is the boundary operator, f(r) is the analytical function and Γ is the boundary domain of Ω. Separating A into two components of linear and nonlinear terms L and N, respectively, Equation (17) is reconstructed as

$$L(u) + N(u) - f(r) = 0 \quad r \in \Omega \tag{19}$$

Homotopy perturbation structure takes the following form:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{20}$$

where

$$v(r, p) : \Omega \times [0, 1] - R \tag{21}$$

P ∈ (0, 1) is the embedding parameter and U<sub>0</sub> is taken as the initial term that satisfies boundary condition. The power series of Equation (21) can be expressed as

$$v = v_0 + p v_1 + p^2 v_2 + p^3 v_3 + \dots \tag{22}$$

Most appropriate solution for the problem takes the following form:

$$u = \lim_{p \rightarrow 1} (v_0 + p v_1 + p^2 v_2 + p^3 v_3 + \dots) \tag{23}$$

**Application of the Homotopy Perturbation Method**

The homotopy perturbation method is the preferred analytical scheme for providing

approximate solutions to the nonlinear ordinary differential. The velocity, heat transfer and concentration series solution may be presented in the following form. Taking power series of velocity, temperature and concentration fields yields

$$F = p^0 f_0 + p^1 f_1 + p^2 f_2 + \dots \quad (24)$$

$$\theta = p^0 \theta_0 + p^1 \theta_1 + p^2 \theta_2 + \dots \quad (25)$$

$$\phi = p^0 \phi_0 + p^1 \phi_1 + p^2 \phi_2 + \dots \quad (26)$$

Substituting Equation (24) into Equation (10) and selecting at the various order yield:

$$p^0 : \frac{d^2 f_0}{d\eta^2} \quad (27)$$

$$p^1 : \frac{d^2 f_1}{d\eta^2} + N \frac{Ra}{Pe} \frac{d\phi_0}{d\eta} + \frac{Ra}{Pe} \frac{d\theta_0}{d\eta} + F \frac{d^2 f_0}{d\eta^2} \frac{df_0}{d\eta} \quad (28)$$

$$p^2 : \frac{d^2 f_2}{d\eta^2} + N \frac{Ra}{Pe} \frac{d\theta_1}{d\eta} + \frac{Ra}{Pe} \frac{d\theta_1}{d\eta} + F \frac{d^2 f_0}{d\eta^2} \frac{df_1}{d\eta} + F \frac{df_0}{d\eta} \frac{d^2 f_1}{d\eta^2} \quad (29)$$

with appropriate boundary conditions, given as

$$f_0(0) + 2M \frac{d\theta_0}{d\eta}(0) = 0, \frac{df_0}{d\eta}(\infty) = 1 \quad (30a)$$

$$f_1(0) + 2M \frac{d\theta_1}{d\eta}(0) = 0, \frac{df_1}{d\eta}(\infty) = 0 \quad (30b)$$

$$f_2(0) + 2M \frac{d\theta_2}{d\eta}(0) = 0, \frac{df_2}{d\eta}(\infty) = 0 \quad (30c)$$

Substituting Equation (25) into Equation (11) and selecting at the various order yield,

$$p^0 : \frac{d^2 \theta_0}{d\eta^2} \quad (31)$$

$$p^1 : \frac{d^2 \theta_1}{d\eta^2} + \frac{4R}{3} \frac{d^2 \theta_0}{d\eta^2} + \frac{f_0}{2} \frac{d\theta_0}{d\eta} + D \frac{df_0}{d\eta} \frac{d^2 \theta_0}{d\eta^2} + D \frac{d\theta_0}{d\eta} \frac{d^2 f_0}{d\eta^2} \quad (32)$$

$$p^2 : \frac{d^2 \theta_2}{d\eta^2} + \frac{4R}{3} \frac{d^2 \theta_1}{d\eta^2} + \frac{f_1}{2} \frac{d\theta_0}{d\eta} + \frac{f_0}{2} \frac{d\theta_1}{d\eta} + D \frac{df_0}{d\eta} \frac{d^2 \theta_1}{d\eta^2} + D \frac{df_1}{d\eta} \frac{d^2 \theta_0}{d\eta^2} + D \frac{d\theta_0}{d\eta} \frac{d^2 f_1}{d\eta^2} + D \frac{d\theta_1}{d\eta} \frac{d^2 f_0}{d\eta^2} \quad (33)$$

The boundary conditions are given as follows:

$$\theta_0(0) = 0, \theta_0(\infty) \rightarrow 1 \quad (34a)$$

$$\theta_1(0) = 0, \theta_1(\infty) \rightarrow 0 \quad (34b)$$

$$\theta_2(0) = 0, \theta_2(\infty) \rightarrow 0 \quad (34c)$$

Substituting Equation (26) into Equation (12) and selecting at the various order yield,

$$p^0 : \frac{d^2 \phi_0}{d\eta^2} \quad (35)$$

$$p^1 : \frac{d^2 \phi_1}{d\eta^2} + BLe \frac{d^2 f_0}{d\eta^2} \frac{d\phi_0}{d\eta} + \frac{Le}{2} \frac{d\phi_0}{d\eta} f_0 + BLe \frac{df_0}{d\eta} \frac{d^2 \phi_0}{d\eta^2} \quad (36)$$

$$p^2 : \frac{d^2 \phi_2}{d\eta^2} + BLe \frac{d^2 f_1}{d\eta^2} \frac{d\phi_0}{d\eta} + BLe \frac{d^2 f_0}{d\eta^2} \frac{d\phi_1}{d\eta} + \frac{Le}{2} \frac{d\phi_0}{d\eta} f_1 + \frac{Le}{2} \frac{d\phi_0}{d\eta} f_0 + BLe \frac{df_0}{d\eta} \frac{d^2 \phi_1}{d\eta^2} + BLe \frac{df_1}{d\eta} \frac{d^2 \phi_0}{d\eta^2} \quad (37)$$

And the boundary conditions are given as

$$\phi_0(0) = 0, \phi_0(\infty) \rightarrow 1 \quad (38a)$$

$$\phi_1(0) = 0, \phi_1(\infty) \rightarrow 0 \quad (38b)$$

$$\phi_2(0) = 0, \phi_2(\infty) \rightarrow 0 \quad (38c)$$

On solving Equations (27), (31) and (35) using the boundary condition in Equations (30a), (34a) and (38a), respectively, we arrived at the following equations:

$$f_0 = \eta - \frac{(2m)}{s} \quad (39)$$

$$\theta_0 = \frac{\eta}{s} \quad (40)$$

$$\phi_0 = \frac{\eta}{s} \quad (41)$$

Similarly, solving Equations (28), (32) and (36) using the boundary condition in Equations (30b), (34b) and (38b), respectively, yields the following equations:

$$f_1 = \frac{m(-s^2 + 6m)}{6s} - \frac{(Ra\eta(\eta - 2s)(N + 1))}{2pes} \tag{42}$$

$$\theta_1 = \frac{-(\eta(\eta - s)(s^2 + \eta s - 6m))}{12s^2}$$

$$\phi_1 = \frac{-Le(\eta(\eta - s)(s^2 + \eta s - 6m))}{12s^2}$$

The coefficient  $p^2$  and the subsequent orders for  $f(\eta)$ ,  $\theta(\eta)$  and  $\phi(\eta)$  in Equations (29), (33) and (37) are too long to be included in this paper. However, they are included in the results which are expressed in graphical and tabular forms. It should be noted following the procedure of HPM that the final expressions for flow, heat transfer and concentration profile for the mixed convective heat transfer flow are given as

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) \tag{45}$$

$$\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) \tag{46}$$

$$\phi(\eta) = \phi_0(\eta) + \phi_1(\eta) + \phi_2(\eta) \tag{47}$$

**Table 1.** Comparison of values for dimensionless concentration ( $\phi'$ ).

M	$\frac{Ra_x}{Pe_x}$	Cheng and Lin [3]	Hemelatha et al. [28]	Present work
0	-0.2	0.5270	0.5269	0.5267
	-0.4	0.4866	0.4866	0.4865
	-0.6	0.4421	0.4421	0.4420
	-0.8	0.3917	0.3917	0.3917
	-1.0	0.3321	0.3321	0.3321

### RESULTS AND DISCUSSION

The results of the developed approximate analytical solutions are compared with the results of the previous works in literature as shown in Table 1. From Table 1, it is verified that HPM gives accurate solution

to the problem. Hence, the approximate analytical solutions are used to investigate the effect of rheological parameters on flow, heat transfer and concentration. The graphical displays are in Figures 1–8.

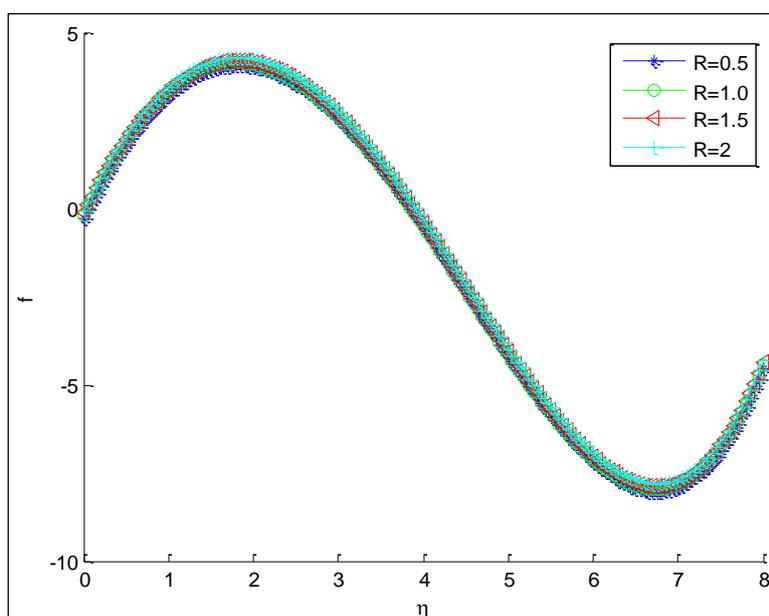
Figure 2 illustrates the effect of radiation parameter ( $R$ ) on the velocity profile. It is depicted that quantitative increase in  $R$  parameter causes a corresponding decrease in velocity distribution whose effect is high toward the lower plate and low toward the upper plate, which can be physically explained by the rapid decrease in mean absorption coefficient effect (44) while effect of melting parameter is demonstrated in Figure 3, which shows decreasing rate of flow as melting parameter ( $M$ ) increases due to increase in Forchheimer empirical constant, with the lowest effect toward the upper plate. Thermal dispersion parameter effect on temperature profile is observed in Figure 3. It is seen that increasing thermal dispersion parameter ( $D$ ) temperature distribution decreases as a result of decreasing thermal dispersion parameter whose effect is maximum toward plate center approximately  $\eta = 4$  (not accurately measured).

It is observed in Figure 4 that the quantitative increase in melting parameter ( $M$ ) increases the temperature distribution in the fluid. This can be physically explained owing to concentration decrease toward solid wall which causes increase in temperature distribution with high effect toward the plate center. Figure 5 depicts the effects of radiation parameter ( $R$ ) on the temperature distribution; it is observed that as  $R$  increases, temperature distribution decreases significantly along the center of flow  $\eta = 4$  (not accurately observed) owing to increasing mean temperature of the flow.

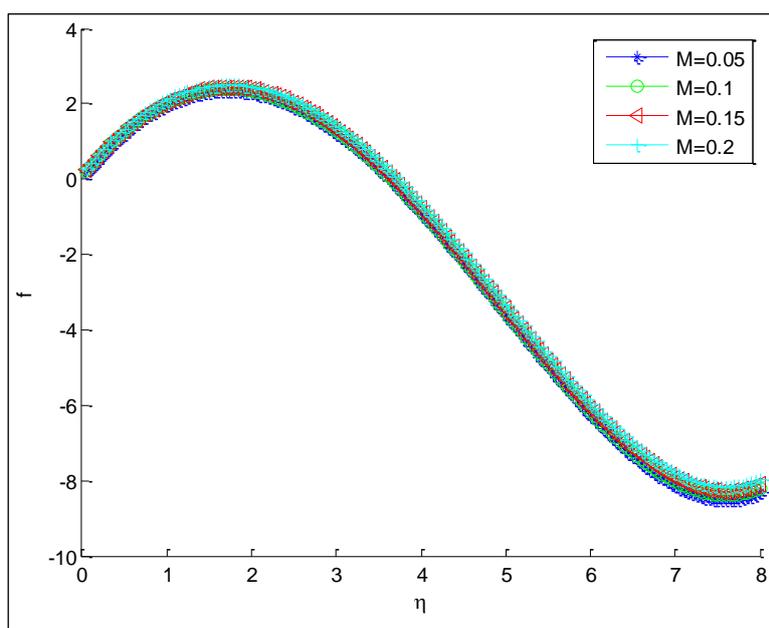
The effect of increasing melt parameter ( $M$ ) on the concentration profile is observed in Figure 6, which illustrates

increasing  $M$  parameter leads to increase in concentration distribution which is significantly high toward the plate center due to corresponding increase of the Forchheimer empirical constant. As effect of increasing Lewis number ( $Le$ ) depicts increase in concentration as observed in Figure 7 as a result of decreasing effect of fluid thermal diffusivity whose effect is maximum at the upper plate.

Also, the solute dispersion parameter ( $B$ ) influence on concentration is presented in Figure 8, which shows numeric increase in  $B$  parameter causes a corresponding decrease in concentration of fluid. Effect is observed to be significant toward plate center. This phenomenon is explained from the physical point of view due to decreasing thermal diffusivity effect as a result of fluid flow.



**Fig. 1.** Effect of radiation parameter on velocity profile.



**Fig. 2.** Effect of melting parameter ( $M$ ) on velocity profile.

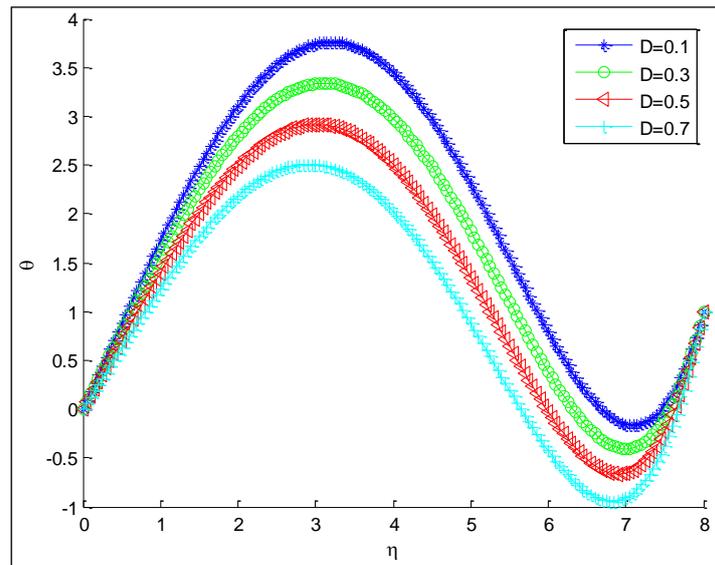


Fig. 3. Effect of thermal dispersion parameter ( $D$ ) on temperature profile.

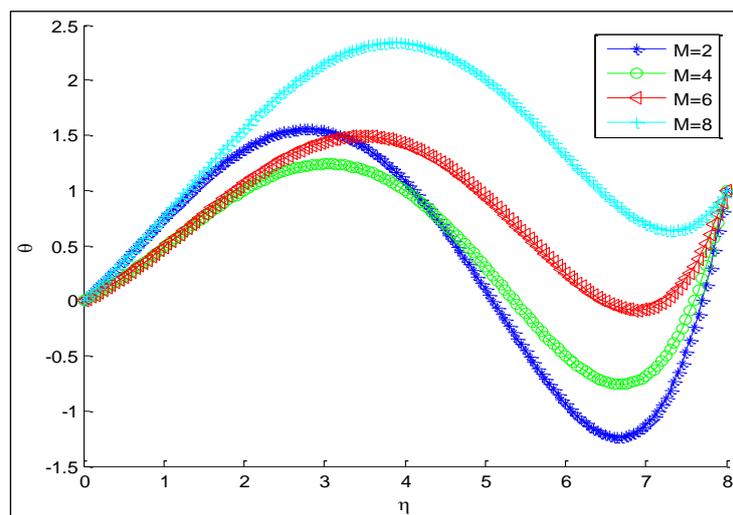


Fig. 4. Effect of melting parameter ( $M$ ) on temperature profile.

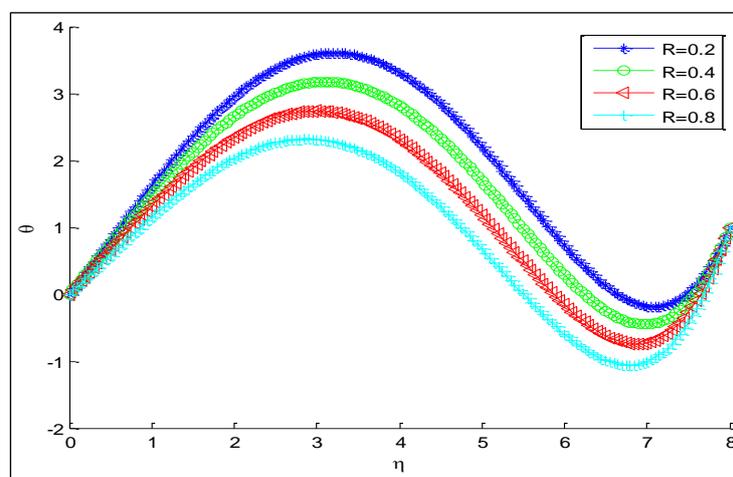


Fig. 5. Effect of radiation parameter ( $R$ ) on temperature profile.

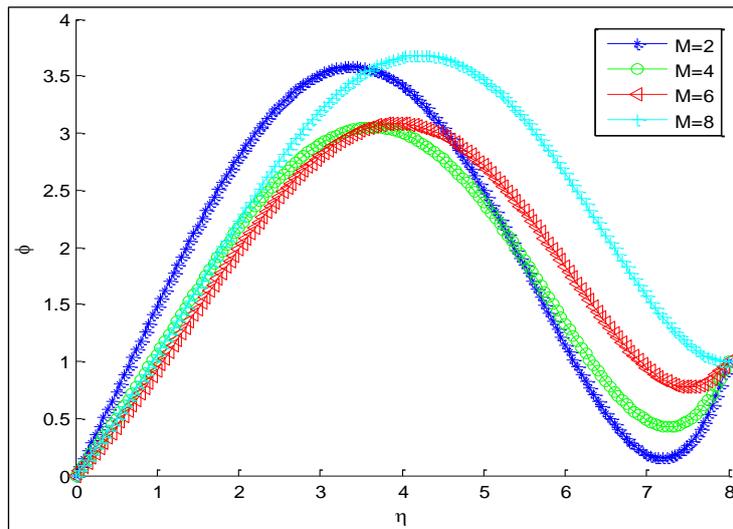


Fig. 6. Effect of melting parameter ( $M$ ) on concentration profile.

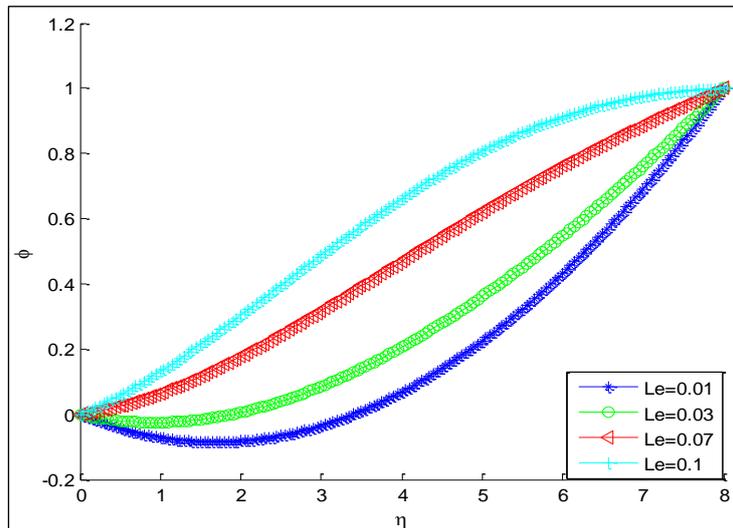


Fig. 7. Effect of Lewis number ( $Le$ ) on concentration profile.

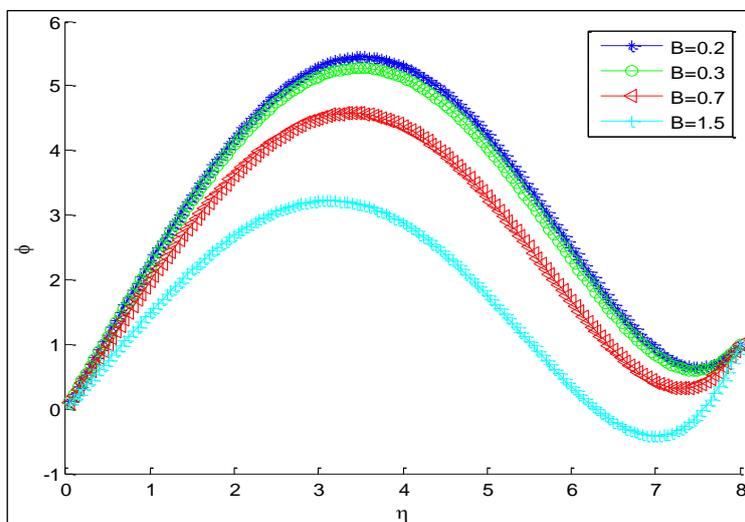


Fig. 8. Effect of solute dispersion parameter ( $B$ ) on velocity profile.

## CONCLUSION

In this study, effects of melting, thermal radiation and dispersion on mixed convective flow, heat and mass transfer through a vertical plate have been investigated using homotopy perturbation method. The obtained approximate analytical solutions generated from complex higher order nonlinear fluid models are used to investigate thermal–fluidic parameters effect on heat transfer and flow through the non-Darcy porous medium. The developed approximate analytical solutions are used to investigate the effect of rheological parameters such as melting, thermal radiation and dispersion parameters. From the results, it is established that as the melting parameter increases, the temperature distribution toward the center plate increases significantly while the concentration distribution increases toward the upper plate as the melt parameter increases. Also, it was observed that increase in  $R$  parameter causes a corresponding decrease in velocity distribution. Increasing thermal dispersion parameter ( $D$ ) decreases temperature distribution. The present study provides useful insight to applications including fossil fuel combustion, astrophysical flows, geothermal systems, material science and metallurgy amongst other applications.

## NOMENCLATURE

$u$  Velocity component in  $x$ -direction  
 $v$  Velocity component in  $y$ -direction  
 $C_f$  Forchheimer empirical constant  
 $\nu$  Kinematic viscosity  
 $g$  Acceleration due to gravity  
 $K$  Permeability of porous medium  
 $C_p$  Specific heat at constant pressure  
 $q_r$  Radiative heat flux  
 $D_e$  Solute diffusivity  
 $F$  Non-Darcy parameter  
 $Ra/Pe$  Mixed convection parameter

$N$  Buoyancy parameter  
 $R$  Radiation parameter  
 $Le$  Lewis number  
 $D$  Thermal dispersion parameter  
 $B$  Solute dispersion parameter  
 $M$  Melting parameter  
 $D_1$  Mass diffusivity  
 $k^*$  Mean absorption coefficient

## Greek Symbols

$\sigma^*$  Stefan–Boltzmann constant  
 $\rho$  Density  
 $\beta_T$  Thermal expansion coefficient  
 $\beta_C$  Solute expansion coefficient

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