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Mathematical Model Development for Turbine Blade Vibration Analysis

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Abstract

Accurate prediction of the natural frequency of tapered steam turbine blade is of considerable importance at the design stage of turbo machines to avoid any resonant conditions leading to the consequent failure of blade due to fatigue. In the present work mathematical modelling of the turbine blade is presented while considering potential energy and kinetic energy. The applications of steam turbine blades and gas turbine blades are increasing with time as the power generation industry is growing at a very fast rate. Therefore, computational analysis at design stage is helpful to minimize the cost of design and in return cost of production.

Keywords: body forces, complementary energy, strain energy, surface traction, variational functional

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INTRODUCTION

A lot of different studies have been performed in power plant system developments. Dev et al[1] developed a computational methodology for the assessment of dual pressure non-reheat combined-cycle power plant with change in drum pressure. A GTA based methodology was also proposed by Dev et al [2] for the evaluation of combined cycle power plant. Thermodynamic analysis of a combined heat and power system was carried out to study the effect of different parameters on power plant performance by Dev et al. [3]

Attri et al.[4,9,21,42,43,57] developed methodology based upon ISM and graph theory for the performance evaluation of different types of the systems. Dev et al. [5, 6] developed a methodology for the analysis of combined cycle power plant performance based upon the combination of graph theory and matrix method. For the performance evaluation CCPP system was divided into six sub-systems. [7, 8] These sub-systems were developed in such a way that all of them were interdependent.[9,10] In literature it is also reported that the system structure developed for the performance evaluation for a power plant or some other type of organization can be extended for the evaluation of other performance parameters.[11,12]

The decision making methodology developed for performance evaluation of combined cycle power plant was extended for the evaluation of reliability and efficiency. [13-15] Dev et al. [16] proposed that reliability is dependent upon the sub-systems. It is the reliability of individual components which affect the reliability of macro-system. Therefore, it is rudiment to consider all of the sub-systems and their interlinking while evaluating the reliability of any system such as combined cycle power plant.[17-19] In that work the methodology developed for decision making for combined cycle power plants was explained with the help of two examples. In those examples it was demonstrated that how the effect of environment could be incorporated in the system. [20-22]

For a power generation organization fuel consumption is one of the foremost criteria of efficiency evaluation. [23-25] It is stressed in literature to develop a methodology for the efficiency evaluation while considering all of the parameter and their interdependence and methodology was based upon the GTA and matrix method. [26-28] In that work, the results obtained with graph theoretic methodology, were in line with the thermodynamic analysis of the power plant. The methodology developed in that work was extended for the performance evaluation of other types of the systems. [29-31]

As the efficiency and reliability both were calculated with the help of same methodology, therefore, it was possible to integrate the results obtained for the efficiency and reliability. [32-35] As a result a common methodology was proposed for the efficiency and reliability evaluation. Problem of decision making is prevailing in every kind of industry or organization. [36-38] In decision making problems a lot of options are available with the managers. Each and every organization is having many of its strength and weaknesses. [39-41]Decision about any problem is based upon these strength and weaknesses.

GTA has been used by many researchers for decision making in the different field of science and technology. [42-44] In literature many different techniques are suggested for the decision making but GTA is different from other decision making techniques in the way of quantifying the inheritance and interdependencies. [44-50] Dev et al (2014) proposed that reliability of a gas turbine system or combined cycle power plant system is dependent upon their sub-systems. The number of sub-systems is dependent upon the mathematical complexity of the analysis. In GTA higher number of sub-systems lessens the complexity of analysis. It was further proposed that reliability of individual components affect the reliability of macro-system that is gas turbine system or combined cycle power plant system. [50-53] Mathematical Modelling for the tapered cantilever beam is presented in the next section.

MATHEMATICAL MODELING

Rao and Rao originally give the direction of energy expressions for a general rotating blade. Displacements, strains and stresses: fig (1) & fig (2) shows the coordinate system used for an asymmetric cross-section blade and the general displacement of an arbitrary particle P fig (3). x_1 x_1 and $y_1 y_1$ and $\eta\eta$ and $\xi\xi$ are two sets of coordinate axes passing through the centroid G of cross section while xx and yy are another set of orthogonal axes, parallel to the x_1 x_1 and $y_1 y_1$ system and passing through the shear center, O. It is assumed that the center of flexure and the torsion center are coincident.

Further it is assumed that the blade is mounted on the disc periphery so that the longitudinal axis, zz will be normal to the disc periphery. The $\eta\eta$ axis lies in the plane of disc rotation and the $\xi\xi$ axes is perpendicular to the plane of disc rotation; thus ϕ is the stagger angle r_x and r_y are the coordinates of shear center w.r.t. centroid.

The disc is assumed to rotate at an angular velocity ω. The initial position of P can be described w.r.t. any one of the orthogonal co-ordinate systems as P

$P(x,y,z)$ or $P(x_1,y_1,z_1)$ or $P(\eta,\xi,z)$

Thus

$$x = x_1 + r_x \quad (1)$$

$$y = y_1 - r_y \quad (2)$$

$$\eta = y_1 Cos\phi - x_1 Sin\phi \quad (3)$$

$$\xi = y_1 Sin\phi + x_1 Cos\phi \quad (4)$$

The general displacement of P consists of translations about the xx and yy axes and rotation about center of flexure, O. The particle initially of P (x, y) moves to P_1, due to rotation θ about O resulting in an inward displacement y θ and an outward displacement of x θ. It further moves from P_1 to P_2 by an amount x and P_2 to P' by an amount y. Thus, the displacement in x and y directions are:

$$u_x = x - y\,\theta = x_1 - y_1\,\theta \quad (5)$$

$$u_y = y + x\,\theta = y_1 + x_1\,\theta \quad (6)$$

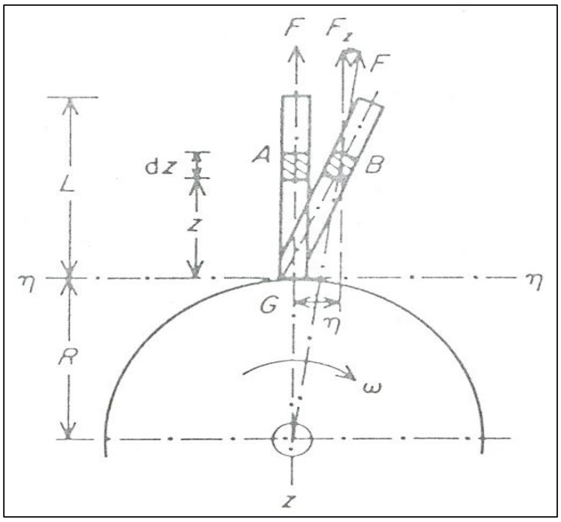

Fig. 1. *Undeflected and Deflected Position Undeflected and Deflected Position of the Blade Mounted on a Rotating.*

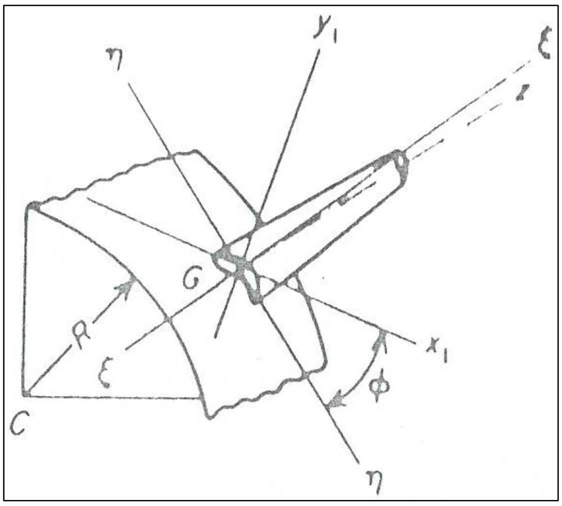

Fig. 2. *Asymmetric Cross-Section Blade Mounted on Disc Periphery) Disc With Deflection in Z-Plane).*

Here

$$x_1 = x + r_y\,\theta \quad (7)$$

$$y_1 = y + r_x\,\theta \quad (8)$$

In above equations, x and y are bending displacements of shear center, O and x_1 and y_1 are bending displacements of centroid, G. Also we can write

$$\mu_\eta = \eta + \theta\xi \quad (9)$$

$$\mu_\xi = \xi = \theta\eta \quad (10)$$

$$\eta = y_1 \cos\phi - x_1 \sin\phi = y \cos\phi - x \sin\phi + a\theta \quad (11)$$

Where $a = r_x \cos\phi - r_y \sin\phi$ (11a)

$$\xi = y_1 \sin\phi + x_1 \cos\phi \quad (12)$$

The particle P after reaching P' will further more in the longitudinal direction due to bending action by an amount ($-x_1 x'_1, - y_1 y'_1$). The longitudinal displacement consists of the contributions due to bending, stretching of the centroidal elements due to the rotation of the disc, and warping. The warping function gives an additional contribution of $\phi_c\,\theta'$ where ϕ_c (x, y) is the warping function, dependent on x, y co-ordinates and independent of 2, then longitudinal displacement is given by

$\mu_z = - \overset{x}{\cdot}{}_1 x'_1 - \overset{y}{\cdot}{}_1 y'_1 + \phi_c \theta' + u$ (13)

Where u = longitudinal displacement of the particle on the line of centroid.

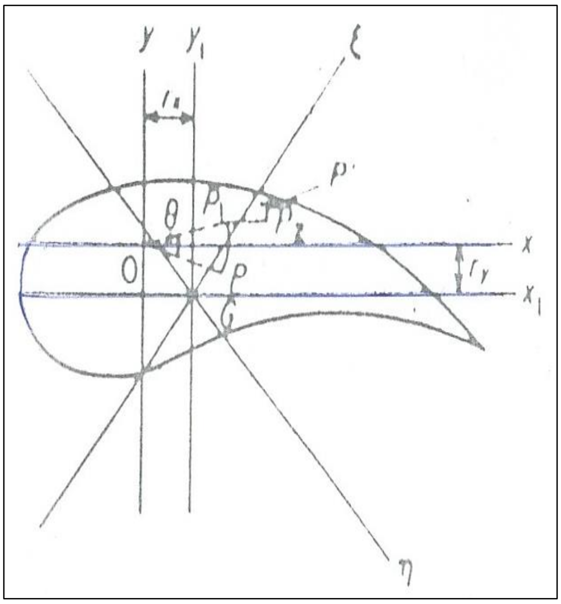

Fig. 3. *Cross-Section of a Blade With Coordinate Axis.*

If the shear deformation effects are to be accounted for, then the displacement in longitudinal direction will be unaffected by the shear slopes so shear slopes are to be eliminated from total slopes in equation (13) and the bending slopes must be used instead of the total slopes.

Thus

$u_z = - x_1 (\phi_1 + \overline{r_y \theta}) - y_1 (\phi_2 + \overline{r_x \theta}) + \phi_c \theta' + u = - x_1 \Psi - y_1 \Psi_2 + \phi_c \phi' + u$ (14)

Where ϕ_1, ϕ_2 = Bending slopes in x_1 and y_1 directions respectively. The strains cannot be calculated from the relation

$\in_{ij} = \frac{1}{2} (u_{ij} + u_{j,i})$ (15)

Therefore using u_x, u_y and u_z, non-zero strains can be written as

$\in_{xz} = \frac{1}{2} \{(x'_1 - \Psi_1) + (\phi_c, x_1 - y_1) \theta'\}$
$= \frac{1}{2} \{(x' - \phi_1) + (\phi_c, x_1 - y_1) \theta'\}$ (16)

$\in_{yz} = \frac{1}{2} \{(y'_1 - \Psi_2) + (\phi_c, y_1 + x_1) \theta'\}$
$= \frac{1}{2} \{(y' - \phi_2) + (\phi_c, y_1 + x_1) \theta'\}$ (17)

$\in_{22} = - x_1 \Psi'_1 - y_1 \Psi'_2 + \phi_c \theta'' + u' - T \alpha$
$= - x_1 \phi'_1 - y_1 \phi'_2 + (\phi_c + x_1 y - y_1 x) \theta'' + u' + t\alpha$ (18)

where $\phi_c, x_1 = \frac{\partial \phi_c}{\partial x_1} etc$

In equation (18) the thermal strain has been taken in to account by assuming T to be steady state temperature distribution. Again in equation (18), The first and second terms ($-x, \phi`1, - y_1 \phi'_2$) represent the fibre extensional strain due to bending slopes alone and the third bracketed term is the strain introduced due to warping and coupling effects.

In deriving equation (16) to (18), the blade is assumed to be straight, uniform and untwisted. The stresses cannot be calculated easily by using equation (16) to (18) in the relation

$T_{ij} = 2 G E_{ij} + e\delta_{ij} \lambda$ (19)

Where λ = Lame's constant = $\gamma E/(1+\gamma)(1-2\gamma)$

A simplified theory can be obtained by assuming the position ratio (γ) to be zero in equation (19). The shear stresses thus calculated are corrected by a shear coefficient k to account for the non-uniform shear stress distribution over the cross section. The non-zero stresses, then are given by

$T_{zx} = KG\{(x' - \phi_1) + (\phi_c, x_1 - y_1)\theta'\} = KG\{(x'_1 - \Psi_1) + (\phi_c, x_1 - y_1)\theta'\}$ (20)
$T_{zy} = KG\{(y' - \phi_2) + (\phi_c, y_1 + x_1)\theta'\} + KG\{(y'_1 - \Psi_2) + (\phi_c, y_1 + x_1)\theta'\}$ (21)
$T_{zz} = E\{-x_1 \Psi'_1 - y_1 \Psi'_2 + \phi_c \theta'' - T\alpha + u'\}$ (22)

Assuming the blade to be untwisted and defining bending moments (M_x, M_y) and shear forces (V_x, V_y) as
$M_x = \int_A T_{zz} y_1 dA = - El_{x1\,y1} \Psi_1 - EI_{x1\,x1} \Psi'_2 - M_{tx}$ (23)
$M_y = \int_A T_{zz} x_1 dA - EI_{y1\,y1} \Psi' - EI_{x1\,y1} \Psi'_2 - M_{ty}$ (24)
$V_x = \int_A T_{zx} dA = kGA(x' - \phi_1) + I\phi_1 G\theta'$ (25)
$V_y = \int_A T_{zy} dA = kGA(y' - \phi_2) + I\phi_2 G\theta'$ (26)

Normal force on a transverse section (N) is given by
$N = \xi_A T_{zz} dA = EA(u' - \alpha t) + EI\phi c\,\theta''$ (27)

Here the quantities $M_{\in x}$ and M_{ty} are thermal bending moments, Ix_1x_1, $I_{x1}\, y_1$, $I\,\phi$ etc. area moments and are given as

$I_{x1\,x1} = \int_A y_1^2 dA$ $I_{y1\,y1} = \int_A x_1^2\, dA$
$I_{x1\,y1} = \int_A x_1 y_1\, dA$
$M_{ex} = \int_A E\alpha T y_1 d\,A$ $M_{t\theta} = \int_A E\alpha T' \phi_c\, dA$
$I\phi_c = \int_A \phi_c dA$ $I\phi_2 = \int_A K(\phi_c, y_1 + x_1) dA$
$I\phi_1 = \int_A K(\phi_c, x_1 - y_1) d\,A$ $I\phi_c\phi_c = \int_A \phi_c^{\,2} d\,A$

In the present case of non-uniform torsion, the net twisting moment consists of the contributions due to the shear stresses T_{zx} and T_{zy}, and also contributions due to the shear stresses developed along with the normal stresses induced by the non-uniform warping, which will become clear once strain energy expression is written in next section. Timoshenko showed by means of reciprocity theorem that the remittent moments due to normal stresses induced by non-uniform torsion vanish, that is

$$Ix_1\phi_c = \int_A x_1\phi_c dA = \int_A (x - rx)\phi_c\, d\,A = 0$$
$$Iy_1\phi_c = \int_A y_1\phi_c dA = \int_A (y - ry)\phi_c\, d\,A = 0 \quad (28)$$

These equations in conjunction with equation (1) & (2), give the co-ordinate r_x, r_y once the warping function is chosen as

$r_x = \dfrac{\int_A x\phi_c\, d\,A}{\int_A \phi_c\, dA}$, $r_y = \dfrac{\int_A y\phi_c\, d\,A}{\int_A \phi_c\, dA}$

VARIATIONAL FUNCTIONAL

The variational functional is defined as

$$I_R = \iiint_v \left[T_{ij} \in_{ij} - U_0^*(T_{ij})\right] dv - \iiint_v B_i \dot{u}_i \, dv - \iiint_{s_i} T_i u_i \, d\,s \qquad (29)$$

$\iiint_v T_{ij} \in_{ij} dv$ = Contribution of strain energy

$\iiint_v U_0^*(T_{ij}) dv$ = contribution of complementary energy

$\iiint_v B_i u_i \, d\nu$ = energy associated with body forces

$\iint_{s_i} T_i u_i \, d\,s$ = energy associated with surface tractions

Since complementary energy functional has to be written in terms of the stresses, the variational functional permits independent variations of both the stresses and the displacement simultaneously which leads to a better approximation of both of these fields.

Strain Energy Functional

$$\iiint_v T_{ij} \in_{ij} d\nu = \int_0^L \iint_A (T_{zz} \in_{zz} + 2T_{zx} \in_{zx} + 2\,T_{zy} \in_{zy})\, dA\, dZ$$

On substituting the strain $\in_{ij}$ from equation (16) to (18) in above equation and performing necessary calculus, we get

$$\int_v T_{ij} \in_{ij} d\nu = \int_0^L \{-M_y \phi'_1 - M_x \phi'_2 + v_x (x' - \phi_1) + b_y) (y' - \phi_2) + T_1\theta' - T_2 \theta''' + G\, I_{\phi c\, x} (x' - \phi_1) \theta' + GI_{\phi c} y (y' - \phi_2) \theta'\} dz$$

Where $T_\theta \theta' = T_1 \theta' - T_2 \theta'' = (c_1 \theta') \theta' - (c_2 \theta') \theta''$

Where $C_1 = KG \int_A \left\{(\phi_{c1} x - y_1)^2 + (\phi c_1 y + x_1)^2\right\} dA$

$C_2 = E \int_A (\phi_c + x_1 y - y_1 x)^2 dA$

Where $T_\theta\theta'$ represent the strain energy due to torsional, thermal and warping twisting moments ($T\theta$). Now neglecting the coupling effects between the bending, warping, torsion and shear terms, we get a simplified expression of follows:

$$\int_v T_{iJ} \in_{iJ} d\nu = \int_0^L \left\{-My\phi_1 - M_x\phi_x + v_x(x' - \phi_1) + v_y(y' - \phi_2) + T_\theta \theta\right\} d\,z \qquad (30)$$

Complementary Energy Functional

None, the stresses given by Equations (20) to (22) are rewritten as follows in terms of equation (23) to (26)

$T_{zx} = (V_x/A) + KG\,\{(\phi c_1\, x_1 - y_1)\,\theta' - (I_{\phi 1}\,\theta'/KA)\}$

$T_{zy} = (V_y/A) + KG\,\{(\phi c_1\, y_1 - x_1)\,\theta' - (I_{\phi 2}\,\theta'/KA)\}$

$$T_{zz} = -x_1 \left\{ \frac{(M_y + y) I_{x_1 x_1} - (M_x + x) I_{x_1 y_1}}{\left(\pm I_{x_1 x_1} I_{y_1 y_1} - I^2_{x_1 y_1} \right)} \right\} - y_1 \left\{ \frac{(M_x + x) I_{y_1 y_1} - (M_y + y) I_{x_1 y_1}}{\left(I_{x_1 x_1}, I_{y_1 y_1} - I^2_{x_1 y_1} \right)} \right\} + E\phi_c \theta''$$

Substituting these equations in complementary energy functional & performing necessary integration by parts, we get

$$\int_v U_0^{*}(T_{iJ})\,dv = \int_0^L \left[\frac{M_x^{\,2} I_{y_1 y_1} - 2 M_x M_y I_{x_1 y_1} + M_y^{\,2} I_{x_1 x_1}}{2E\left(I_{x_1 x_1} I_{y_1 y_1} - I^2_{x_1 y_1}\right)} + \frac{c_2 (\theta')^2}{2} \right.$$

$$+ \left. \frac{v_x^{\,2} + v_y^{\,2}}{2KGA} + \frac{C_1(\theta)^2}{2} - \frac{G(\theta)^2}{2KA} - \left(I_\phi^2 cx + I\phi_{c\,y}^2 \right) \right] dz$$

Further making the assumption that the coupling terms of warping, bending, torsion and shear are negligible we get

$$\int_v V_0^{*}(T_{iJ})\,dv = \int_0^{2} \left[\frac{M_x^2 I_{y_1 y_1} - 2 M_x M_y + M_y^2 I_{x_1 x_1}}{2E\left(I_{x_1 x_1}, I_{y_1 y_1} - I^2_{x_1 y_1}\right)} + \frac{T_\theta^2}{2c} + \frac{vx^2}{2c} + \frac{v_x^2 + x_y^2}{2KGA} \right] dz \qquad (31)$$

Body Forces and Surface Tractions

The gravitational and thermal forces as well as the inertia forces developed due to rotation of blade – disc assembly contributes to the body forces that are distributed over the volume of the blade material. Generally the effect of gravitational forces on a non-rotating blade can be ignored by measuring the energies associated with the vibration of the blade about the static equilibrium position, and the corresponding procedure is understood to be applied here for rotating blade. The surface fractions that are likely to be present in the turbo machine blade include the aerodynamic forces and moments, steam or gas bending pressure or a host of other phenomena due to the flow of the fluid relative to the moving or stationary blade moves. These forces may be unsteady but for the sake of simplicity and generalization, such external forces are assumed to be steady and thus being independent of time, can be represented by the intensities of distributed load and moment, L_x, L_y and M, acting through or about the elastic axis. The work done by these external forces is then given by

$$\iint_{s_1} T_i u_i\, ds = \int_0^L \left(L_x x + L_y y + L\theta \right) dz \qquad (32)$$

And a large number of coupling term between warping & now by neglecting body forces & surface tractions shear and further assuming that blade is untwisted and that the energies associated with asymmetry are negligible and substituting equation (30) & (31) in equation (29) we get variational functional as

$$I_R = -\int_0^L M_x \phi_2 + M_y \phi_1 - v_x (x' - \phi_1) - v_y (y' - \phi_2) - T_\theta \theta +$$

$$\frac{Mx^2 I_{y_1y_1} - 2M_xM_yI_{x_1x_1}}{2E\left(I_{x_1x_1}I_{y_1y_1} - I^2_{x_1y_1}\right)} + \frac{T\theta^2}{2c} + \frac{V_x^2 + v_y^2}{2KGA}dz \quad (33)$$

Equation (33) gets amplified further if we assume asymmetry in yy – direction only, in which case x and y deflections get uncoupled and $I_{x1\ y1}$ will vanish because of the symmetry in one plane and also $\begin{pmatrix} My = 0, \\ vx = 0 \end{pmatrix}$

The resulting expression is

$$I_R = -\int_0^L \left[M_x\phi_2^1 - v_y\left(y' - \phi_2\right) - T_\theta\theta + \frac{Mx^2}{eEI_{x_1x_1}} + \frac{T\theta^2}{2C} + \frac{v_y^2}{2KGA}\right] dz \quad (34)$$

Kinetic Energy

The Kinetic energy of a blade mounted on a disc rotating at an angular velocity on consists of the contributions due to translational and longitudinal inertias and also the contributions due to centripetal and Coriolis effects. The gain in kinetic energy due to centripetal effects is given by

$$T_{ce} = \int_0^L\int_A \frac{\delta w^2}{2}\left(\eta + u_\eta\right)^2 + (R + z + u_z)^2]\, d\,A\, d\,z$$

Where R = Disc radius and δ = mass density

The gain in kinetic energy due to Coriolis effects is

$$T_{c0} = \int_0^L\int_A \delta w\left[u_\eta\left(R + z + u_z\right)^2\right] dA dz$$

The gain in kinetic energy due to Coriolis effects is

$$T_{c0} = \int_0^L\int_A \delta w\, [u_\eta\,(R + z + u_z) - (\eta + u_\eta)\, u_z]\, dA\, dz$$

The kinetic energy due to translational and longitudinal inertias is

$$T_i = \frac{1}{2}\iiint_v \delta \dot{u}_i \dot{u}_i\, dv = \frac{1}{2}\iiint_v \delta\left[\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_2^2\right] dv = \frac{1}{2}\int_0^L\int_A \delta\left[\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_2^2\right] dAdz$$

Now as we are considering only static deflection of blades to neglecting K.E's due to centripetal effects and Coriolis effects. K.E. is given by

$$T = \frac{1}{2}\int_0^L\int_A \left[\delta\left(\dot{u}_x^2 + \dot{u}_y^2 + \dot{u}_z^2\right)\right] dAd. =$$

$$\frac{1}{2}\int_0^L [m\dot{x}_1^2 + m\dot{y}_1^2 + mp\dot{\theta}^2 + m\dot{u}^2 + m\dot{x}x_1(\dot{\phi}_2 + r_x\dot{\theta})^2 + m\dot{y}y_1(\dot{\phi}_1 + r_y\dot{\theta})^2 + m_{\phi_c\phi_c}\left(\dot{\theta}\right)^2\ ]dZ$$

None if one assumes symmetry in one plane only i.e. $r_y = 0$, and neglecting effect of warping, we get K.E. as

$$T = \int_0^L \left[\frac{\delta A}{2}(y+r_x\theta)^2 + \frac{\delta Ip}{2}\theta^2 + \frac{\delta I_{x_1x_1}}{2}(\phi_2 + r_x\theta)^2 \right] dz$$

Here m = δA, mp = δIp, m_{x1x1} = δ $_{Ix1x1}$, x_1 = (y + r_x θ)

So if one assumes symmetry in one plane only i.e. r_y = 0, one obtained coupled bending – torsion vibrations in (y - θ) and uncoupled flexural vibrations in x z – plane. For this particular case with ϕ = 90^0, the dynamic variation functional L_R can be written by subtracting equation (34) from (35) for the coupled y - θ vibrations as

$L_R = T - I_R$

$$L_R = \int_0^L \left[\begin{aligned} & \frac{\delta A}{2}(y+r_x\theta)^2 + \frac{\delta Ip}{2}\theta^2 + \frac{\delta I x_1 x_1}{2}\left(\theta_2 + r_x\theta\right) + M_x\phi_2 \\ & -V_y\left(y' - \phi_2\right) - T_\theta\theta + \frac{Vy^2}{2KGA} + \frac{M_x^2}{2EI_{x_1x_1}} + \frac{T\theta^2}{2c} \end{aligned} \right] dz \quad (36)$$

The time averaged value of the dynamic variational functional can be given by

$$I_R = \int_0^{2\pi/p} (T - I_R)\, dt$$

$$\Rightarrow I_R = \int_0^{2\pi/p}\int_0^L \left[\begin{aligned} & \frac{\delta A}{2}(y+r_x\theta)^2 + \frac{\delta Ip}{2}\theta^2 + \frac{\delta I_{x_1x_1}}{2}(\phi_2 + r_x\theta)^2 + M_x\phi_2 \\ & -vy(y' - \phi_2) - T_\theta\theta + \frac{V_y^2}{2KGA} + \frac{Mx^2}{2EI_{x_1x_1}} + \frac{T_\theta^2}{2C} \end{aligned} \right] dz\, dt \quad (37)$$

CONCLUSION

In the present work a methodology is presented for the prediction of vibrations in tapered cantilever beam. Accurate prediction of the natural frequency of tapered steam turbine blade is of considerable importance at the design stage of turbo machines to avoid any resonant conditions leading to the consequent failure of blade due to fatigue.

In the present work mathematical modelling of the turbine blade is presented while considering potential energy and kinetic energy. The applications of steam turbine blades and gas turbine blades are increasing with time as the power generation industry is growing at a very fast rate. Therefore, computational analysis at design stage is helpful to minimize the cost of design and in return cost of production.

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