International Journal of Robotics and Automation
Vol. 1: Issue 1
www.journalspub.com

Estimation of Optimum Robot Heading Using Savitzky-Golay and Kalman Filters

Anand Krishnamoorthy and Girish Kumar
Department of Mechatronics, SRM University, Kattankulathur, India

Abstract

Determination of robot heading angle is imperative in mobile robot navigation and localization. The raw sensor data from MEMS IMUs are noisy and prone to drifting. This paper presents a methodology for constructing an optimum estimate for the heading angle by employing two filters—Savitzky-Golay and Kalman filter to fuse magnetometer and gyroscope data. For the evaluation of the proposed method, a mobile robot was constructed with a control board consisting of MEMS IMUs and Arduino controller. The experimental results illustrate the performance of the filters for noisy sensor measures.

Keywords: *Savitzky-Golay filter, kalman filter, magnetometer, heading angle, sensor fusion*

****Author for Correspondence:*** *Email ID: anand_krishnamoorthy@srmuniv.edu.in*

INTRODUCTION

Localization and navigation are obligatory for mobile robots. Localization consists of the determination of the azimuth orientation of the robot and its position in a given reference frame. The best method to determine the heading angle is from the compass which measures from the magnetic north of the Earth[1, 2]. But the performance characteristics of the MEMS magnetometers and gyroscopes are limited by error sources. To eliminate these errors, filtering techniques are required. Various methods are currently used to estimate the robot heading angle from MEMS sensors. Robot heading can be processed by fusing gyroscopes and odometric sensors using constrained Kalman filter[3]. Encoder and gyroscope are also fused in dead reckoning navigation of autonomous robots using indirect Kalman filter[4]. Other variations of the Kalman filter used in sensor fusion for robot localizations include, enhanced particle Kalman filter (EPKF)[5], extended Kalman filter (EFK), iterated extended Kalman filter (IEKF)[6], federated Kalman filter[7] and unscented Kalman filter[8]. Neural network-based sensor fusion algorithms have also been developed for robot localization[9]. In this paper, the application of a statistical filter-Savitzky-Golay filter is proposed for filtering the magnetometer measure. The magnetometer gives the instantaneous values and has low signal-to-noise ratio (SNR). Moreover, the heading angle is determined from the *x* and *y* axes data of the magnetometer in the sensor's reference frame. This technique does not assure uniform increment of the angle. Because of the unbounded growth of errors in MEMS systems, they are often required to be coupled with other sensors. These are done by sensor fusion algorithms. This paper presents Kalman filter for the aforementioned procedure. This paper presents a description of Savitzky-Golay filter and the Kalman filter followed by the implementation routine of these filters and the sensor fusion architecture proposed. The hardware structure of the system used for the evaluation is described later.

FILTER ALGORITHMS

Savitzky-Golay Filter

The Savitzky-Golay filter is a statistical digital filter which uses least squares method (LSM) to fit a discrete set of points to a polynomial curve of degree $'d'$[10]. It is a finite impulse response (FIR) low pass filter used for smoothening data. The LSM is proven to be the most optimal method, provided the noise distribution in the signal is *Gaussian*.

The Savitzky-Golay filter sets coefficients C_i for the data array y of length n, and fits to a polynomial of degree d, given by the equation:

$$Y_j = (C \otimes y) = \sum_{i=\frac{-(d-1)}{2}}^{i=\frac{(d-1)}{2}} C_i y_{i+j} \qquad \text{Eq. (1)}$$

The midpoint value of the convolution array Y gives the filtered output at that time instant. The coefficients are chosen such that the mean square error (MSE) given by,

$$\epsilon_i = \frac{1}{n}\sum_{i=1}^{i=n}(Y_i - y_i)^2 \qquad \text{Eq. (2)}$$

is least at the center point. Thus to calculate the output at any other instant of time, the new data is added to the array and is shifted so that the reconstruction is done at the midpoint value. This process is called *data centering* [11]. One requirement of this filter is that the index of the filter, i.e., the window length of the data array must be an odd positive integer. The interval must be symmetric about the midpoint for *linear* filters. In case of nonlinear filters, the smoothing happens at the ends of the window. A generalized form of Savitzky-Golay filter is a moving average filtering with a least square fit at each data point. The main characteristic of this filter as compared to other statistical filters is that smoothing is achieved without loss of resolution.

Kalman Filter

The Kalman filter is a technique developed from estimation theory that combines the information of different uncertain sources to obtain the values of interest with a lower standard deviation as compared to the source information. Since the true state of the system cannot be determined, the Kalman filter provides a technique to estimate the state based on the combination of the system model and the noisy linear functions on parameters. It is derived from the mean square estimator commonly used in linear models. It exploits the property of *Gaussian* distribution of noise—the convolution of two *Gaussians* is another *Gaussian*.

The Kalman filter addresses the general problem of estimating a state vector x at the instant $k+1$ governed by the linear stochastic difference equation:

$$x_{k+1} = F_k x_k + B_k u_k + w_k \qquad \text{Eq. (3)}$$

With a measurement vector y_k

$$y_k = H_k x_k + z_k \qquad \text{Eq. (4)}$$

Where, F_k, B_k represents the state transition matrix and the control input matrix, respectively. H_k is the observation matrix and w_k, z_k are process noises and the observation noise, respectively. The discrete Kalman filter estimates the state using a feedback control at every discrete time instant. The estimate of the previous time instant feeds back in the form of noisy measurement. Thus the Kalman filter consists of two parts—the time update and the measurement update.

After each cycle of time and measurement updates, the *posteriori* states of the previous measurement is used to predict the *priori* states. The Kalman filter recursively conditions the current estimate based on its past estimates. The main significance of this filter as compared to other data fusion filters such as Weiner filter is its ability to accommodate nonstationary data and noises.

FILTER ARCHITECTURE

Savitzky-Golay Filter

Savitzky-Golay filter is used to increase the SNR of the magnetometer data. It is reasonable to assume the noise in the magnetometer data to be *Gaussian*. The incoming magnetometer data is stored in an array of length n which represents the window of this filter. The coefficient array obtained from the polynomial of degree d, is convolved with the magnetometer array to obtain the filtered heading.

Selection of Optimal Filter Parameters

The minimum index of $d+1$ is required to fit a polynomial of degree a. If the index is less than the degree of polynomial, the equations required for solving by least squares (LS) method becomes inconsistent, and if equal to the degree, then it replicates the input data and no smoothing occurs. If the degree is too high, then over fitting occurs which leads to poor estimation of data. Thus, to select the optimal parameters required for our purpose, we chose a maximum and minimum window length and iterated the process by arithmetic progression of 2 satisfying the symmetry requirements [12]. The MSE and the frequency response of the filter were taken to be the criteria for evaluation. Table 1 gives the filter design for the parameters under consideration.

The optimal parameters for our filter were of a cubic polynomial degree and a window data length of 9 and their characteristic plots are shown in Figures 1 and 2.

Table 1: *Savitzky-Golay Filter Test Parameters.*

Normalised Savitzky-Golay coefficients					
Polynomial order = 3			**Polynomial order = 5**		
n = 7	**n = 9**	**n = 11**	**n = 7**	**n = 9**	**n = 11**
-0.0952	-0.0909	-0.0839	0.0216	0.0349	0.0419
0.1428	0.0606	0.0209	-0.1298	-0.128	-0.1048
0.2857	0.1688	0.1025	0.3246	0.0699	-0.0233
0.3333	0.2337	0.1608	0.5670	0.3146	0.1398
0.2857	0.2554	0.1958	0.3246	0.4172	0.2797
0.1428	0.2337	0.2074	-0.1298	0.3146	0.3333
-0.0952	0.1688	0.1958	0.02164	0.0699	0.2797
	0.0606	0.1608		-0.128	0.1398
	-0.0909	0.1025		0.0349	-0.0233
		0.0209			-0.1048
		-0.0839			0.0419

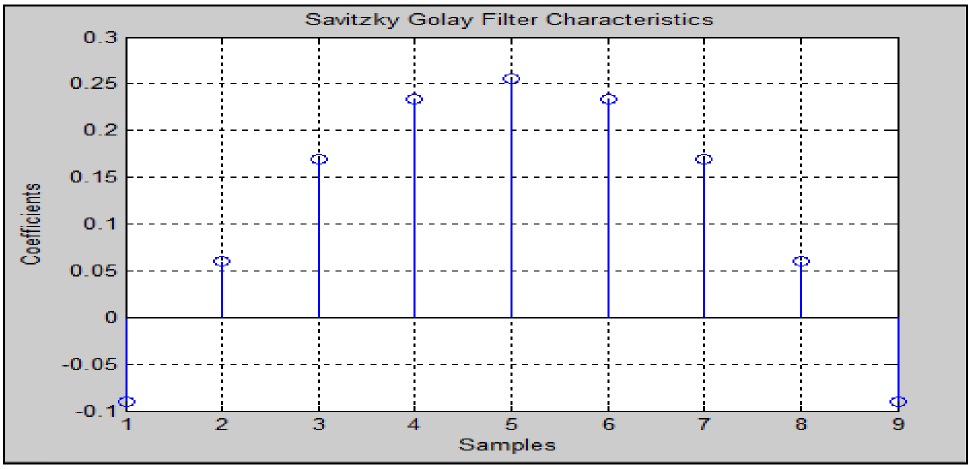

Fig.1: *Savitzky-Golay Filter Characteristics–Time Domain.*

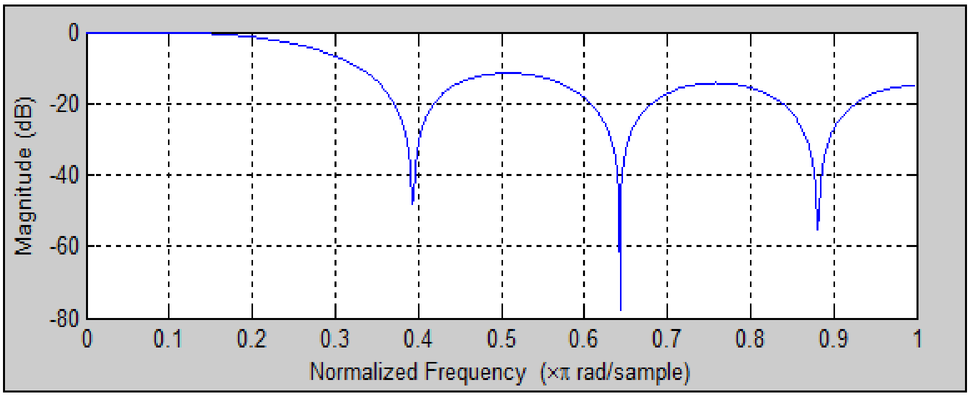

Fig. 2: *Frequency Response of the Savitzky-Golay Filter.*

Kalman Filter and Sensor Fusion

The state vector x of the Kalman filter is defined by two arguments—the heading angle from the magnetometer and the z-rate from the gyroscope. These measurements include a component of error, i.e., has some arbitrary standard deviation. Thus the task of Kalman filter is to combine the values of magnetometer and the gyroscope to produce a heading which is as close to the true value. The Kalman implementation can be summarized as follows:

Obtain the priori state and the error covariance matrix from the previous state; obtain the measurement and the innovation matrix from the observation matrix; obtain the posteriori state and the error covariance matrix from the updated Kalman gains. The process control flow for Kalman filter is described below:

Time Update

Step 1: Estimate the current state vector from the previous states

$X_k = FX_{k-1} + B\dot{\theta};$ Eq. (5)

Where, $\dot{\theta}$ is the gyroscope measurement

Step 2: Update the process error covariance matrix from the previous state

$P_k = FP_{k-1}F + Q_k\Delta t$ Eq. (6)

Measurement Update

Step 3: Compute the measurement residue

$y_k = Z_k - HX_k$ Eq. (7)

Where, Z_k is the measured angle from magnetometer

Step 4: Compute the innovation matrix

$S_k = HP_kH^T + R$ Eq. (8)

Step 5: Update the Kalman gains

$K_k = P_kH^TS^{-1}$ Eq. (9)

Step 6: Update the estimate of the state vector from the current state

$X_{k+1} = X_k + K_ky_k$ Eq. (10)

Step 7: Update the process error covariance matrix for the next state

$P_{k+1} = (I - K_kH)P_k$ Eq. (11)

Where,

$F -$ State transition matrix;

$B -$ Control input model;

$P -$ Error covariance matrix;

$Q -$ Covariance matrix;

$H -$ Observation matrix;

$K -$ Kalman gain matrix;

$S -$ Innovation matrix;

$R -$ Measurement noise

i. Optimal Parameters for Kalman Filter

For the given state vector

$x = \begin{bmatrix}\varphi \\ \dot{\varphi}\end{bmatrix},$ Eq. (12)

Where, $\dot{\varphi}, \varphi$ are the yaw rate and the heading angle.

The state transition matrix F is given by,

$$F = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \quad \text{Eq. (13)}$$

The control vector matrix is given by,

$$B = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \quad \text{Eq. (14)}$$

Since the measurement is the heading from the magnetometer, the observation matrix is given by,

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad \text{Eq. (15)}$$

The initial parameters were chosen based on iterations for optimality. The resultant covariance matrix Q and the measurement noise R were set as,

$$Q = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.003 \end{bmatrix}; R = 0.01 \quad \text{Eq. (16)}$$

Sensor Fusion

Sensor fusion is the process of combining external sensor data to represent the same quantity with a lower error variance. The sensors are viewed as different members of a class whose output is a function of the congruent data. Kalman filter is the most widely used data fusion algorithms currently and has better performance for linear data as compared to the standard Bayesian filter. Due to the stochastic nature of noise, Kalman filter is almost the predominant technique used for sensor fusion [13].

There are two broad types of sensor integration—*tightly* coupled and *loosely* coupled. In *loosely* coupled sensor integration, the yaw angle from the gyro is preprocessed by an internal Kalman filter using the magnetometer heading prior to error correction.

In *tightly* coupled, the filtered data of the magnetometer from the Savitzky-Golay filter is used in the correction of the gyro yaw angle. For our evaluation, a tightly coupled integration was used. Figure 3 describes the control flow of the data for the filter architecture.

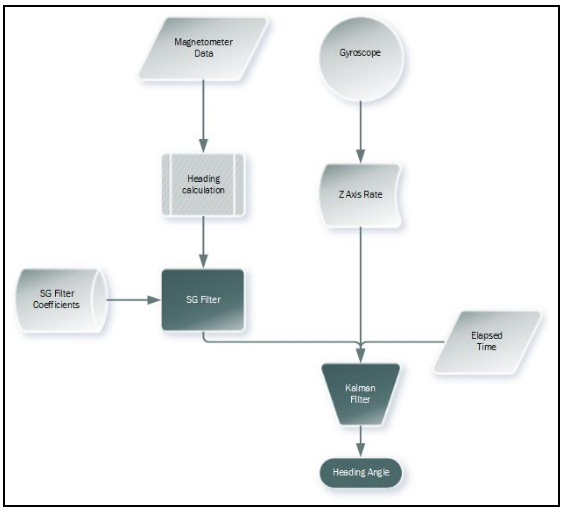

Fig. 3: *Flowchart of the Filter Architecture.*

HARDWARE DESCRIPTION

The hardware used for the evaluation of the proposed filter architecture was MPU9150 MEMS IMU. Its magnetometer is a three-axis silicon monolithic Hall-effect magnetic sensor with magnetic concentrator and has a resolution of 13 bits (Figure 4) [14].

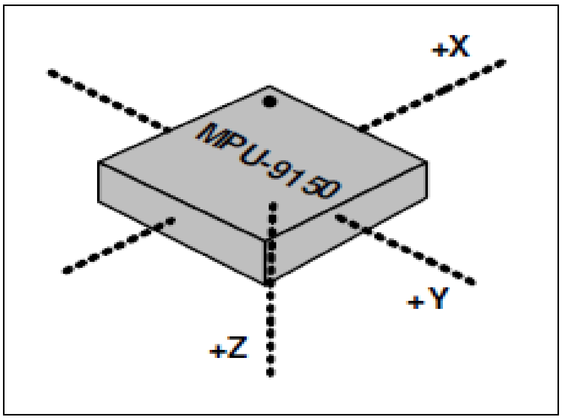

Fig. 4: *MPU 9150 Magnetometer Axes Configuration.*

The heading angle φ from the magnetometer is calculated by the following equation:

$$\varphi = atan2(mag_y, mag_x) \quad \text{Eq. (17)}$$

Where, mag_x and mag_y are the x and y axis values of the magnetometer, respectively; and $atan2$ is defined as,

$$atan2(x,y) = \begin{Bmatrix} arctan\frac{x}{y}; x > 0 \\ arctan\frac{x}{y} + \pi; y \geq 0, x < 0 \\ arctan\frac{x}{y} - \pi; y < 0, x < 0 \\ \frac{\pi}{2}; y \geq 0, x = 0 \\ -\frac{\pi}{2}; y < 0, x = 0 \end{Bmatrix}$$

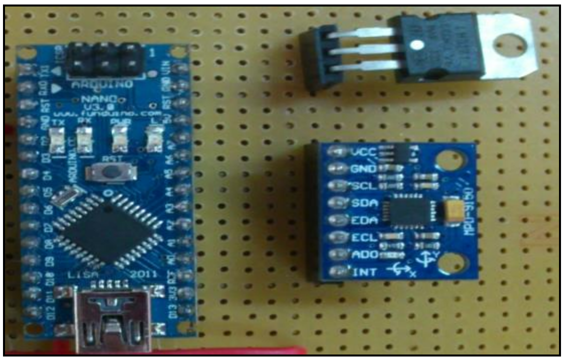

Fig. 5: *Evaluation Board with MPU 9150 Interfaced Arduino Nano.*

The Arduino Nano® board was used as the controller and the MPU 9150 was interfaced to it (Figure 5). The data from the IMU were serially transmitted though serial protocol interface (SPI) at 100 Hz. The filter architecture was coded into the Atmega 328 microcontroller and was also processed at 100 Hz. The mobile robot was set to wander about the setup arena and the raw and filtered values were recorded.

RESULTS

In our experimental implementation, the output heading value from the filter architecture was greatly dependent on the initial calibration of the magnetometer and the gyroscope. The SNR of the input data from the magnetometer was calculated to be 1.435. Table 2 presents the evaluation results from the test parameters of the Savitzky-Golay filter.

The optimal Savitzky-Golay parameters were chosen not only based on the filter characteristics but also on the execution time as well as the program memory required. The chosen stated cubic polynomial fit with an index of 9 was tested to be efficient for out particular application and requirements (Figure 6).

Table 2: *Evaluation Results.*

Savitzky-Golay parameters	Execution time (ms)	Memory required(Bytes)	SNR
$p = 3, n = 7$	20.41	80	1.7502
$p = 3, n = 9$	20.48	96	1.7675
$p = 3, n = 11$	20.51	112	1.7701
$p = 5, n = 7$	20.72	96	1.7616
$p = 5, n = 9$	20.77	112	1.7638
$p = 5, n = 11$	20.80	128	1.7656

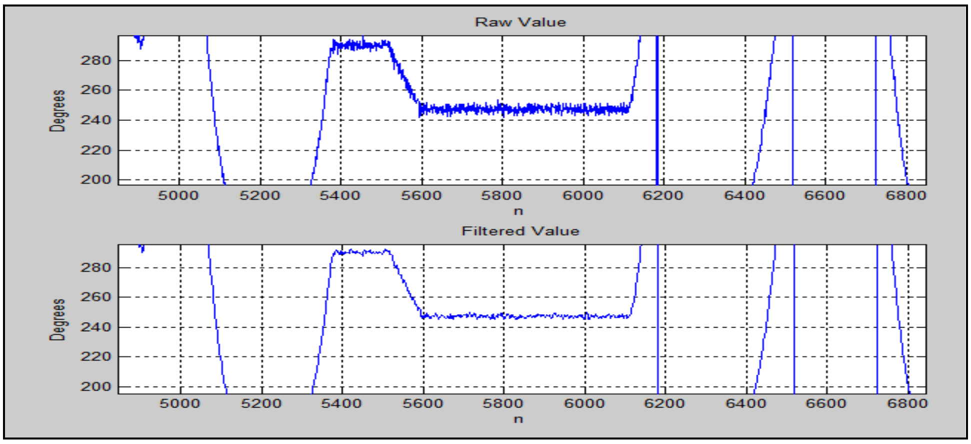

Fig. 6: *Savitzky-Golay Filtering Results.*

It can be observed that the filtered value almost follows the true posture. The performance of the Kalman filter for fusing the filtered magnetometer data and the yaw angle from gyroscope is depicted in Figure 7. The bias and the measurement error was set such that accurate heading angle was obtained. The total execution time was 42.94 msec and the resultant heading had a SNR of 2.1229. The filter architecture yielded smooth and optimum heading data from the noisy magnetometer and drifting gyrosensor measures. Compared to the current gyroscope–odometer data fusion implementations [15–21], the proposed filter architecture produces superior results and is consistent.

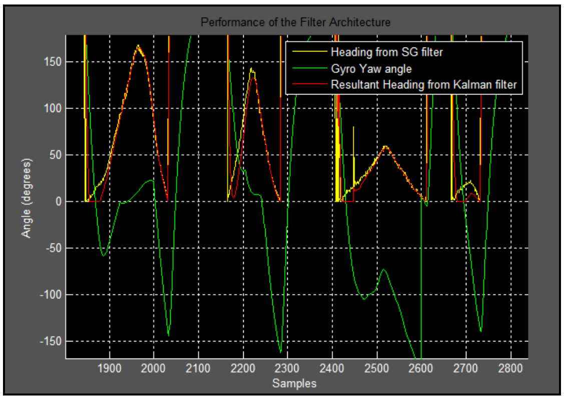

Fig. 7: *Performance of the Savitzky-Golay–Kalman Filter Architecture.*

CONCLUSION

In this paper, the Savitzky-Golay filter and the standard Kalman filter are used for the robot heading estimation. From the plots obtained from the data of the evaluation hardware, the proposed filter architecture is consistently better at producing the optimum heading as compared to the other filtering techniques. The Savitzky-Golay filter was found to be optimum for filtering linear data without loss of resolution and with lesser program memory required for its software implementation. The standard Kalman filter with its optimized parameters produced the robot heading with minimum variance. This filter architecture can also be applied for distance estimation combining the data from accelerometer and GPS; thus in general for a generic robot localization and navigation applications.

ACKNOWLEDGEMENT

We thank our guide Mr. J. Balaji, who guided us throughout this work and provided us with critical feedback. This work was done as a part of the final year project "Development of autonomous swarm robots based on alpha-beta coordination" under his guidance at the SRM University, Kattankulathur, India.

REFERENCES

1. Renaudin V., Afzal M.H., Lachapelle G. New Method for Magnetometers Based Orientation Estimation. *Proceedings of the IEEE/ION Position, Location and Navigation Symposium (PLANS) 2010*; 2010 May 4–6; Palm Springs, CA, USA. 348–56p.
2. Skvortzov V.Y., Lee H.K., Bang S.W., *et al*. Application of Electronic Compass for Mobile Robot in an Indoor Environment. *Proceedings of the IEEE International Conference on Robotics and Automation, 2007*; 2007 Apr 10–14; Roma, Italy. 2963–70p. DOI: 10.1109/ROBOT.2007.363922
3. Myung H., Lee H.K., Choi K., *et al*. Mobile Robot Localization Using a Gyroscope and Constrained Kalman Filter. *Proceedings of the SICE-ICASE International Joint Conference 2006*; 2006 Oct 18–21; Bexco, Busan, Korea.
4. Park K.C., Chung D., Chung H., *et al*. Dead Reckoning Navigation of a Mobile Robot Using an Indirect Kalman Filter. *Proceedings of the 1996 IEEE/SICE/RSJ International Conference on Multisensor Fusion and Integration for Intelligent Systems*; 1996 Dec 8–11; Washington, DC, USA. 132–9p.
5. Mahmoud II, Tawab A.A.E., Salama M. An Enhanced Particle / Kalman

Filter for Robot Localization. *Proceedings of the 30th National Radio Science Conference (NRSC 2013)*; 2013 Apr 16–18; Cairo, Egypt. 338–46p.
6. Kleeman L. Optimal estimation of position and heading for mobile robots using ultrasonic beacons and dead-reckoning. IEEE *International Conference on Robotics and Automation*; 1992 May 10–15; Nice, France. 2582–7p.
7. Zhang Q., Qiu X., Xu G., *et al*. Azimuth Estimation of Mobile Robot Based on Federated Kalman Filter. *Proceedings of the 2010 International Conference on Computer, Mechatronics, Control and Electronic Engineering (CMCE)*; 2010 Aug 24–26; Changchun, China. 202–6p.
8. Anjum M.L., Park J., Hwang W., *et al*. Sensor Data Fusion using Unscented Kalman Filter for Accurate Localization of Mobile Robots. *Proceedings of the International Conference on Control, Automation and Systems 2010;* 2010 Oct 27–30; Gyeonggi-do, Korea. 947–52p.
9. Axenie C., Conradt J. Cortically Inspired Sensor Fusion Network for Mobile Robot Heading Estimation. *Artificial Neural Networks and Machine Learning—ICANN 2013. Lecture Notes in Computer Science*. 2013; 8131: 240–7p.
10. Luo J., Ying K., Bai J. Savitzky–Golay smoothing and differentiation filter for even number data. *Elsevier Signal Processing*. 2005; 85(7):1429–34p. DOI:10.1016/j.sigpro.2005.02.002 b)
11. Schafer R.W. What Is a Savitzky-Golay Filter? [Lecture Notes]. *IEEE Signal Processing Magazine*. 2011; 28(4): 111–7p. DOI: 10.1109/MSP.2011.941097
12. Krishnan S.R., Seelamantula C.S. On the Selection of Optimum Savitzky-Golay Filters. *IEEE Transactions on Signal Processing*. 2013; 61(2): 380–91p. DOI: 10.1109/TSP.2012.2225055
13. Park K.C., Chung H., Choi J., *et al*. Dead reckoning navigation for an autonomous mobile robot using a differential encoder and a gyroscope. *Proceedings of the 8th International Conference on Advanced Robotics, 1997 (ICAR '97)*; 1997 Jul 7–9; Monterey, CA, USA. 441–6p. DOI: 10.1109/ICAR.1997.620219
14. InvenSense Inc. *MPU-9150 Product Specification Revision 4.2.* Document Number: RM – MPU -9150A – 00. Release Date: 09/18/2013. Available from: http://www.invensense.com/mems/gyro/mpu9150.html
15. Houshangi N., Azizi F. Mobile Robot Position Determination Using Data Integration of Odometry and Gyroscope. *Proceedings of the World Automation Congress, 2006 (WAC '06)*; 2006 Jul 24–26; Budapest, Hungary. 1–8p. DOI: 10.1109/WAC.2006.375994
16. Wei Y., Lee M. A new MEMS Gyro north finding approach using LSM for mobile robot heading detection. *Proceedings of the SICE Annual Conference 2012 (SICE)*; 2012 Aug 20–23; Akita, Japan. 2262–7p.
17. Jintao Y. The design of mobile robot heading control. *2010 International Conference on E-Health Networking, Digital Ecosystems and Technologies (EDT)*; 2010 Apr 17–18; Shenzhen, China. 9–11p. DOI: 10.1109/EDT.2010.5496513
18. Hara S., Ishimoto T. A least squares-based heading direction estimation for a robot swarm with only ranging capability. *Proceedings of the IEEE International Workshop on Robotic and Sensors Environments, 2009 (ROSE 2009);* 2009 Nov 6–7; Lecco, Italy. 1–5p. DOI: 10.1109/ROSE.2009.5355998
19. Hargittai S. Savitzky-Golay least-squares polynomial filters in ECG

signal processing. *Proceedings of the Computers in Cardiology, 2005*; 2005 Sep 25–28; Lyon.763–6p. DOI: 10.1109/CIC.2005.1588216

20. Gander W., Matt U.V. Smoothing Filters. In: Gander W, Hrebicek J (Eds.). *Solving Problems in Scientific Computing Using Maple and MATLAB®.* Berlin, Heidelberg: Springer-Verlag; 1997. 135–52p.
21. Jingwei S., Yongjie Z., Haiyun Z., *et al*. A multi-MEMS sensors information fusion algorithm. *Proceedings of the 26th Chinese Control and Decision Conference (2014 CCDC)*; 2014 May 31 Jun 2; Changsha, China. 4675–80p. DOI: 10.1109/CCDC.2014.6853008
22. Chung H., Ojeda L., Borenstein J. Sensor fusion for mobile robot dead-reckoning with a precision-calibrated fiber optic gyroscope. *Proceedings of the 2001 IEEE International Conference on Robotics and Automation*; 2001 May 21–26; Seoul, South Korea. 3588–93p. DOI: 10.1109/ROBOT.2001.933174
23. Maeyama S., Ishikawa N., Yuta S. Rule based filtering and fusion of odometry and gyroscope for a fail safe dead reckoning system of a mobile robot. *Proceedings of the IEEE/SICE/RSJ International Conference on Multisensor Fusion and Integration for Intelligent Systems, 1996;* 1996 Dec 8–11; Washington, DC, USA. 541–8p. DOI: 10.1109/MFI.1996.572228
24. Hong S.K., Ryuh Y.S. Heading Measurements for Indoor Mobile Robots with Minimized Drift using a MEMS Gyroscopes. In: Hanafiah Y (Ed.). Robot Localization and Map Building. In Tech; 2010. ISBN 978-953-7619-83-1 DOI: 10.5772/9263