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Passivity Analysis of Uncertain Stochastic Neural Networks with Discrete and Distributed Time-Varying Delays

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Abstract

This paper investigates the problem of robust passivity of uncertain stochastic neural networks with discrete and distributed time-varying delays. To reflect the most dynamical behaviors of the system, both parameter uncertainties and stochastic disturbance are considered, where parameter uncertainties enter into all the system matrices, stochastic disturbances are given in the form of a Brownian motion. By utilizing the Lyapunov functional method, the Itô differential rule and matrix analysis techniques, we establish sufficient criterion such that, for all admissible parameter uncertainties and stochastic disturbances, the stochastic neural networks is robustly passive in the sense of expectation. The delay - dependent stability condition is formulated, in which the restriction of the derivative of the time-varying delay should be less than 1 is removed. The derived criteria are expressed in terms of linear matrix inequalities (LMIs) that can be easily checked by using the standard numerical software. Illustrative examples are presented to demonstrate the effectiveness and usefulness of the proposed results.

Keywords: delayed neural networks, linear matrix inequality, lyapunov functional, parameter uncertainty, passivity

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INTRODUCTION

In recent years, neural networks have received considerable attention due to their extensive applications in solving some optimization problem, associative memory, classification of patterns, and other areas. Time delays are unavoidably present due to the finite switching speeds of the amplifiers and the inherent communication time of neurons, and its existence will affect the stability of a network by creating oscillatory and instability characteristics. [13, 14] Therefore, the stability analysis of neural networks with delays has recently received much attention. [1, 10, 19, 27, 20, 23, 25, 26, 28, 32, 40] When one models real nervous systems, stochastic disturbance and parameter uncertainties are unavoidable to be considered. Because in real nervous system, synaptic transmission is a noisy process brought on by random fluctuation from the release of neurotransmitters, [17] and the connection weights of the neuron depend on certain resistance and capacitance values that include uncertainties. Therefore, it is of practical importance to study the stochastic effects on the stability of neural networks with parameter uncertainties, some results related to this problem have been published in the papers mentioned in the references. [2, 7, 16, 18, 36–39]

In addition, parameter uncertainties can be often encountered in real systems as well as neural networks, due to the modelling inaccuracies and/or changes in the environment of the model. In the past few years, to solve the problem brought by parameter uncertainty, robustness analysis for different uncertain systems has received considerable attention. [22, 30, 31, 35]

On another research front, the passivity problem for a variety of practical systems has been attracting renewing attention for many years. The passivity theory was firstly proposed in the circuit analysis [3] and since then has found successful applications in diverse areas such as stability, signal processing, complexity, fuzzy control, chaos control, and synchronization. [8, 11, 24, 33, 41] For instance, the passivity problem of the uncertain neural networks with time varying delays. Very recently, the passivity problems were dealt with stochastic neural networks with time varying delays. [11] To the best of author's knowledge, there are no results on the passivity of stochastic delayed neural networks with discrete and distributed time varying delays and parameter uncertainties.

Motivated by the above discussions, in this paper we aim to investigate the passivity of stochastic delayed neural networks with discrete and distributed time varying delays and parameter uncertainties. A novel Lyapunov functional method combined with the matrix analysis techniques is developed to obtain sufficient conditions under which the system is globally robustly passive in the sense of expectation. These sufficient conditions are expressed in terms of LMIs that can be solving numerically. These conditions are delay-dependent, that is, the conditions depend on the size of time-varying delays. It is usually less conservative than those delay-independent ones. Finally, numerical examples are given to show the effectiveness of the obtained results.

Notations

Throughout the manuscript we will use the notation $A > 0$ to denote that the matrix A is a symmetric and positive definite matrix. Let $(\Omega, F, \{F_t\}_{t\geq 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t\geq 0}$ satisfying the usual conditions (it is right continuous and F_0 contains all P-null sets); $\mathcal{L}^{p}_{F0}([-\tau,\ 0];\ R^n)$ be the family of all bounded, F_0- measurable, $C([-\tau,\ 0];\ R^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} E|\xi(\theta)|^p < \infty$.

The mathematical expectation operator with respect to the given probability measure P is denoted by $E\{\cdot\}$. The shorthand *diag* $\{\cdot\ \cdot\ \cdot\}$ denotes the block diagonal matrix. k · k stands for the Euclidean norm. Moreover, the notation * always denotes the symmetric block in one symmetric matrix.

SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following uncertain stochastic recurrent neural networks with time-varying delays described by,

$$dx(t) = [-(A(t) + \Delta A(t))x(t) + (B(t) + \Delta B(t)))f(x(t)) + (W(t) + \Delta W(t))f(x(t - \tau(t))) + (D(t)+ \tag{1}$$

$$\Delta D(t)) \int_{t-\tau(t)}^{t} f(x(s))ds + J(t)]dt + [(C(t) + \Delta C(t))x(t) + (H(t) + \Delta H(t))x(t - \tau(t))]d\omega(t),$$

$$y(t) = f(x(t)),$$

$$x(t) = \phi(t),\ -\bar{\tau} \leq t \leq 0.$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the neural state vector, *J(t)* is the input, y(t) is the output, τ (t) is the unknown time-varying delay which satisfies $0 \leq \tau(t) \leq \bar{\tau} < \infty$, where $\bar{\tau}$ and d are known constants.

The matrix A(t) is a diagonal matrix and $B(t) \in R^{n\times n}$, W(t), D(t) ∈ R n×n are the connection weight matrices, $C(t) \in R^{n\times n}$ and $H(t) \in R^{n\times n}$ are known real constant matrices, ΔA(t), ΔB(t), ΔW(t), ΔC(t), ΔD(t) and ΔH(t) represents the time-varying parameter uncertainties and are assumed to be of the form:

$$[\Delta A\ \Delta B\ \Delta W\ \Delta C\ \Delta H\ \Delta D(t)] = MF(t)[N_1\ N_2\ N_3\ N_4\ N_5\ N_6], \tag{2}$$

where M, N_1 N_2 N_3 N_4 N_5 and N_6 are known real constant matrices and F(t), are known time-varying matrix functions satisfying

$$F^T(t)F(t) \leq I. \tag{3}$$

It is assumed that all elements F(t) are Lebesque measurable, ΔA, ΔW ΔB, ΔC, ΔD(t) and ΔH are said to be admissible if both (2) and (3) hold.

Further, ω(t) = [ω1(t), ω2(t), . . . , ωm(t)]T ∈ R m is a m− dimensional Brownian motion defined on a complete probability space (Ω, F,P) and f(x(t)) = [f1(x1(t)), f2(x2(t)), . . . , fn(xn(t))]T ∈ R n is the neuron activation function with f(0) = 0. 3 The following assumption is made on the neuron activation function. (A) Each neuron activation functions f(·) in system (1) are bounded and satisfy the following condition 0 ≤ fi(x) − fi(y) x − y ≤ Li , (4) where Li (i = 1, 2 . . . , n) are some constants and they can be positive. So it is less restrictive than the descriptions on both the sigmoid activations and the Lipschitz type activation functions.

Denote L = diag{L1, . . . , Ln}. Defining the following variables for the stochastic neural networks (1) with time varying delays, g1(t) = −(A(t) + ΔA(t))x(t) + (B(t) + ΔB(t)))f(x(t)) + (W(t) + ΔW(t))f(x(t − τ (t))) + (D(t) + ΔD(t)) Z t t−τ(t) f(x(s))ds + J(t), g2(t) = (C(t) + ΔC(t))x(t) + (H(t) + ΔH(t))x(t − τ (t)) Then stochastic neural networks (1) with time varying delays is described as dx(t) = g1(t)dt + g2(t)dω(t), (5) Integrating both sides of (5) from t − τ (t) to t yields x(t) − x(t − τ (t)) = Z t t−τ(t) g1(s)ds + Z t t−τ(t) g2(s)dw(s).

Remark 2.1-The parameter uncertainty structure as in (2) − (3) has been widely exploited in the problems of robust control and robust filtering of uncertain systems (see [30] and the references there in). The stochastic disturbance term, [(C(t) + ΔC(t))x(t) + (H(t) + ΔH(t))x(t − τ (t))]dω(t), can be viewed as stochastic perturbation on the neuron states and delayed neuron states.

Definition 2.2.- The system (1) is said to be globally robustly passive in the sense of expectation if there exists a scalar β ≥ 0 such that E n Z t 0 J T (s)y(s)dso ≥ −βEn Z t 0 J T (s)J(s)dso , for allt ≥ 0 for all admissible uncertainties (3) and (4) and solution x(t, 0) of 1. Before stating the main results, we need to introduce the following notations and lemmas which will be essential for the proof of few results in the next section.

Let C 1,2 (R × Rn, R+) denote the family of all nonnegative function V (t, x) on R × Rn which are continuously twice differentiable in x and once differentiable in t. For each V ∈ C

1,2 (R × Rn, R+), by Itˆo's differential formula, the stochastic derivative of V (t, x(t)) along (1) can be obtained as:
dV (t, x(t)) = LV (t, x(t))dt + Vx(t, x(t))[(C + ΔC(t))x(t) + (H + ΔH(t))x(t − τ (t))]dω(t) where L is the weak infinitesimal operator of the stochastic process {xt = x(t+s)|t ≥ 0, −τM ≤ s ≤ 0}, given by
LV (t, x(t)) = Vt(t, x(t)) + Vx(t, x(t))h − (A + ΔA(t))x(t) + (B + ΔB(t))f(x(t))+ (W + ΔW(t))f(x(t − τ (t))) + (D + ΔD(t)) Z t t−τ(t) f(x(s))ds + J(t) i + 1 2 trace[(C + ΔC(t))x(t) + (H + ΔH(t))x(t − τ (t))]T 4 × Vxx[(C + ΔC(t))x(t) + (H + ΔH(t))x(t − τ (t))] with Vt(t, x(t)) = ∂V (t, x(t)) ∂t , Vx(t, x(t)) = ∂V (t, x(t)) ∂x1 , . . . , ∂V (t, x(t)) ∂xn , Vxx(t, x(t)) = ∂ 2V (t, x(t)) ∂xi∂xj n×n.

Lemma 2.3. (Schur complement [4]). The LMI " Q(y) S(y) S(y) T R(y) # < 0 is equivalent to R(y) < 0, Q(y) − S(y)R(y) −1S(y) T < 0 where Q(y) = Q(y) T , R(y) = R(y) T , and S(y) depend affinely on y.

Lemma 2.4. [15] For any constant matrix P ∈ Rn×n, P = P T > 0, scalar r > 0, and vector function ϕ : [0, r] → Rn, one has r Z r 0 ϕ T (s)P ϕ(s)ds ≥ Z r 0 ϕ T (s)dsT P Z r 0 ϕ T (s)ds , provided that the integrals are well defined.

Lemma 2.5. [15] For given matrices D, E and F with F T F ≤ I and scalar > 0, the following inequality holds, DF E + E T F T DT ≤ DDT + −1EET . Lemma 2.6. [6] Given any real matrices K1, K2, Q of appropriate dimensions and a number ν > 0 such that 0 < Q = QT , then the following inequality holds:
KT 1 K2 + KT 2 K1 ≤ νKT 1 QK1 + ν −1KT 2 Q −1K2. 3.

Global passivity results In this section, some sufficient conditions for passivity of the system (1) without uncertainties are obtained.

Theorem 3.1. The delayed neural networks (1) with ΔA = ΔB = ΔW = ΔC = ΔD(t) = ΔH = 0 is said to be globally passive in the sense of expectation, if there exist symmetric positive definite matrices P > 0, Q > 0, R > 0, S > 0, V > 0, positive diagonal matrices T1 > 0, T2 > 0 and any matrices O1, O2, O3, U, U¯ such that feasible solution exist for the following LMI,

$$\Omega = \begin{bmatrix} \bar{\Omega} & O & S \\ * & -V & 0 \\ * & 0 & -P \end{bmatrix} < 0, \qquad (6)$$

where

$$\bar{\Omega} = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & \Omega_{1,3} & \Omega_{1,4} & \Omega_{1,5} & \Omega_{1,6} & \Omega_{1,7} \\ * & \Omega_{2,2} & \Omega_{2,3} & \Omega_{2,4} & \Omega_{2,5} & \Omega_{2,6} & \Omega_{2,7} \\ * & * & \Omega_{3,3} & \Omega_{3,4} & \Omega_{3,5} & \Omega_{3,6} & \Omega_{3,7} \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & \Omega_{4,6} & \Omega_{4,7} \\ * & * & * & * & \Omega_{5,5} & \Omega_{5,6} & \Omega_{5,7} \\ * & * & * & * & * & \Omega_{6,6} & \Omega_{6,7} \\ * & * & * & * & * & * & \Omega_{7,7} \end{bmatrix}$$

$$\Omega_{1,1} = -PA - A^T P + R + O_1^T + O_1,\ \Omega_{1,2} = -O_1 + O_2^T + A^T \bar{U}^T,$$

$$\Omega_{1,3} = PB + LT_1,\ \Omega_{1,4} = PW,\ \Omega_{1,5} = P\ D,\ \ \Omega_{1,6} = P,\ \Omega_{1,7} = O_3^T - A^T U^T,$$

$$\Omega_{2,2} = -(1-d)R - O_2^T - O_2,\ \Omega_{2,3} = \bar{U}B,\ \Omega_{2,4} = LT_2 + \bar{U}W,\ \Omega_{2,5} = \bar{U}W,$$

$$\Omega_{2,6} = \bar{U},\ \Omega_{2,7} = -O_3^T - \bar{U},\ \Omega_{3,3} = Q + \bar{\tau}^2 S - 2T_1,\ \Omega_{3,4} = 0,\ \Omega_{3,5} = \Omega_{4,6} = 0,$$

$$\Omega_{3,6} = -I,\ \Omega_{3,7} = B^T\ U^T,\ \Omega_{4,7} = W^T\ U^T,\ \Omega_{4,4} = -(1-d)Q - 2T_2,\ \Omega_{4,5} = 0,$$

$$\Omega_{3,7} = B^T\ U^T,\ \Omega_{5,5} = -S,\ \Omega_{5,6} = 0,\ \Omega_{5,7} = D^T\ U^T,\ \ \Omega_{6,6} = -\gamma I,\ \Omega_{6,7} = U^T,$$

$$\Omega_{7,7} = \bar{\tau}^2 V - U^T - U,\ \bar{O}^T = [O_1^T\ O_2^T\ 0\ 0\ 0\ 0\ O_3^T],\ \bar{S}^T = [PC\ PH\ 0\ 0\ 0\ 0\ 0].$$

Proof: We use the following Lyapunov functional to derive the stability result
V (t, x(t)) = V1(t, x(t)) + V2(t, x(t)) + V3(t, x(t)) + V4(t, x(t)), (7)

Where,
V1(t, x(t)) = x T (t) P x(t),
V2(t, x(t)) = Z t t−τ(t) f T (x(s)) Q f(x(s)) ds,
V3(t, x(t)) = Z t t−τ(t) x T (s) R x(s) ds,
V4(t, x(t)) = ¯τ Z 0 −τ¯ Z t t+θ f T (x(s)) S f(x(s)) dsdθ,
V5(t, x(t)) = ¯τ Z 0 −τ¯ Z t t+θ g T 1 (s) V g1(s)) dsdθ.

By Itˆo's formula, the stochastic derivative of V (t, x(t)) along the trajectories of the system (1) with ΔA = ΔB = ΔW = ΔC = ΔD = ΔH = 0 can be obtained as, LV (t, x(t)) = LV1(t, x(t)) + LV2(t, x(t)) + LV3(t, x(t)) + LV4(t, x(t)) + LV5(t, x(t)), (8) where LV1(t, x(t)) ≤ 2x T (t)P h − Ax(t) + Bf(x(t)) + W f(x(t − τ (t)) + D Z t t−τ(t) f(x(s))ds + J(t) i + [Cx(t) + Hx(t − τ (t))]T P[Cx(t) + Hx(t − τ (t))] LV2(t, x(t)) ≤ f T (x(t)) Q f(x(t)) − (1 − d)f T (x(t − τ (t))) Q f(x(t − τ (t))) LV3(t, x(t)) ≤ x T (t) R x(t) − (1 − d) x T (t − τ (t))Rx(t − τ (t)) LV4(t, x(t)) ≤ τ¯ 2 f T (x(t))Sf(x(t)) − τ¯ Z t t−τ(t) f T (x(s))Sf(x(s))ds, LV5(t, x(t)) ≤ τ¯ 2 g T 1 (t)V g1(t) − τ¯ Z t t−τ(t) g T 1 (s)V g1(s)ds. 6 By Jensen's Inequality [15] −τ¯ Z t t−τ(t) f T (x(s))Sf(x(s))ds ≤ −Z t t−τ(t) f(x(s))dsT S Z t t−τ(t) f(x(s))ds, (9) −τ¯ Z t t−τ(t) g T 1 (s)V g1(s)ds ≤ −Z t t−τ(t) g1(s)dsT V Z t t−τ(t) g1(s)ds. (10) For any matrices U, U¯ of appropriate dimensions, it can be shown that 2[x T (t − τ (t))U¯ + g T 1 (t)U] h − g1(t) − Ax(t) + Bf(x(t)) + W f(x(t − τ (t))) + D Z t t−τ(t) f(x(s))ds +J(t) i = 0. (11) 2[x T (t)O1 + x T (t − τ (t))O2 + g T 1 (t)O3][x(t) − x(t − τ (t)) − Z t t−τ(t) g1(s)ds − Z t t−τ(t) g2(s)dw(s)] = 0. (12) By using Lemma 2.6 −2ξ T 1 (t)O Z t t−τ(t) g1(s)ds ≤ ξ T 1 (t)OV −1O T ξ1(t) + Z t t−τ(t) g1(s)dsT V Z t t−τ(t) g1(s)ds (13) where ξ T 1 (t) = [x T (t) x T (t − τ (t)) g T 1 (t)], OT = [OT 1 OT 2 OT 3]. Therefore we have dV (t, x(t)) ≤ 2x T (t)P h − Ax(t) + Bf(x(t)) + W f(x(t − τ (t))) + D Z t t−τ(t) f(x(s))ds + J(t) i + f T (x(t)) Q f(x(t)) + x T (t) R x(t) − (1 − d) x T (t − τ (t))Rx(t − τ (t)) + (1 − d)f T (x(t − τ (t))) Q f(x(t − τ (t))) + ¯τ 2 f T (x(t))Sf(x(t)) − τ¯ Z t t−τ(t) f T (x(s))Sf(x(s))ds − τ¯ Z t t−τ(t) g T 1 (s)V g1(s)ds + ¯τ 2 g T 1 (t)V g1(t) + 2[x T (t − τ (t))U¯ + g T 1 (t)U][−g1(t) − Ax(t) + Bf(x(t)) + W f(x(t − τ (t))) + J(t) i + 2[x T (t)O1 + x T (t − τ (t))O2 + g T 1 (t)O3][x(t) − x(t − τ (t)) − Z t t−τ(t) g1(s)ds − Z t t−τ(t) g2(s)dw(s) + [Cx(t) + Hx(t − τ (t))]T P[Cx(t) + Hx(t − τ (t))] × +2x T (t)P[Cx(t) + Hx(t − τ (t))]dw(t) o . (14) From assumption (A) we know that f T (x(t))[f(x(t)) − Lx(t)] ≤ 0, f T (x(t − τ (t)))[f(x(t − τ (t))) − Lx(t − τ (t))] ≤ 0. Then, for T1 = diag{t11, . . . , t1n} ≥ 0, T2 = diag{t21, . . . , t2n} ≥ 0 we have dV (t) ≤ dV (t) − 2T1f T (x(t))[f(x(t)) − Lx(t)] 7 − 2T2f T (x(t − τ (t)))[f(x(t − τ (t))) − Lx(t − τ (t))]. (15) Finally, from (8)-(15), we have dV (t, x(t)) − γJT (t)J(t) − J T (t)y(t) ≤ ξ T (t) Ξ ξ(t) + ξ T 1 (t)OV −1O T ξ1(t) − 2ξ T 1 (t)O Z t t−τ(t) g2(s)dw(s) + 2x T (t)P h Cx(t) + Hx(t − τ (t))i dw(t) o , (16) where ξ(t) = [x T (t) x T (t − τ (t)) f T (x(t)) f T (x(t − τ (t))) Z t t−τ(t) f(x(s))dsT J T (t) g T 1 (t)]^T

$$\xi(t) = [x^T(t) \;\; x^T(t-\tau(t)) \;\; f^T(x(t)) \;\; f^T(x(t-\tau(t))) \;\; \Big(\int_{t-\tau(t)}^{t} f(x(s))ds\Big)^T \;\; J^T(t) \;\; g_1^T(t)]^T,$$

$$\Xi = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} & \Xi_{1,3} & \Xi_{1,4} & \Xi_{1,5} & \Xi_{1,6} & \Xi_{1,7} \\ * & \Xi_{2,2} & \Xi_{2,3} & \Xi_{2,4} & \Xi_{2,5} & \Xi_{2,6} & \Xi_{2,7} \\ * & * & \Xi_{3,3} & \Xi_{3,4} & \Xi_{3,5} & \Xi_{3,6} & \Xi_{3,7} \\ * & * & * & \Xi_{4,4} & \Xi_{4,5} & \Xi_{4,6} & \Xi_{4,7} \\ * & * & * & * & \Xi_{5,5} & \Xi_{5,6} & \Xi_{5,7} \\ * & * & * & * & * & \Xi_{6,6} & \Xi_{6,7} \\ * & * & * & * & * & * & \Xi_{7,7} \end{bmatrix},$$

$\Xi_{1,1} = -PA - A^TP + R + C^TPC + O_1^T + O_1,\ \Xi_{1,2} = C^TPH - A^T\bar{U}^T - O_1 + O_2^T,\ \Xi_{1,4} = PW,$

$\Xi_{1,5} = PD,\ \ \Xi_{1,6} = P,\ \ \Xi_{1,7} = O_3^T - A^TU^T,\ \Xi_{2,2} = H^TPH - (1-d)R - O_2 - O_2^T,$

$\Xi_{2,3} = \bar{U}B,\ \Xi_{2,4} = LT_2 + \bar{U}W,\ \Xi_{2,5} = \bar{U}D,\ \Xi_{2,6} = \bar{U},\ \Xi_{2,7} = -O_3^T - \bar{U},\ \Xi_{3,3} = Q + \bar{\tau}^2S - 2T_1,$

$\Xi_{3,4} = \Xi_{3,5} = \Xi_{4,5} = \Xi_{4,6} = 0,\ \Xi_{3,6} = -I,\ \Xi_{3,7} = B^TU^T,\ \Xi_{4,4} = -(1-d)Q - 2T_2,\ \Xi_{1,3} = PB + LT_1,$

$\Xi_{4,7} = W^TU^T, \Xi_{5,5} = -S,\ \Xi_{5,6} = 0,\ \Xi_{5,7} = D^TU^T,\ \Xi_{6,6} = -\gamma I,\ \Xi_{6,7} = U^T,\ \Xi_{7,7} = \bar{\tau}^2V - U - U^T.$

By using Schur complement lemma we have dV (t, x(t)) − γJT (t)J(t) − J T (t)y(t) ≤ ξ T (t) Ω ξ(t) − 2ξ T 1 (t)O Z t t−τ(t) g2(s)dw(s) + 2x T (t)P h Cx(t) + Hx(t − τ (t))i dw(t) o , (17) Taking the mathematical expectation on the both sides of (17), one can deduce that E{dV (t, x(t)) − γJT (t)J(t) − J T (t)u(t)} ≤ E(ξ T (t)Ωξ(t)) ≤ 0, which means E{ Z t 0 J T (s)y(s)ds} ≥ E V1(t, x(t)) − V1(t, 0) − γ Z t 0 J T (s)J(s)ds = E V1(t, x(t)) − γ Z t 0 J T (s)J(s)ds ≥ −γE Z t 0 J T (s)J(s)ds .

From Definition 2.2, we know that the stochastic neural network (1) with ΔA = ΔB = ΔW = ΔC = ΔD = ΔH = 0 is globally passive in the sense of expectation, and the proof of Theorem 3.1 is completed.

8 4. Robust passivity results In this section, some sufficient conditions for passivity of the system (1) are obtained.

Theorem 4.1. The delayed neural networks (1) is said to be globally passive in the sense of expectation, if there exist symmetric positive definite matrices P > 0, Q > 0, R > 0, S > 0, V > 0, positive diagonal matrices T1 > 0, T2 > 0 any matrices O1, O2, O3, U, U¯ and positive scalars 1, 2 such that feasible solution exist for the following LMI,

$$\Pi = \begin{bmatrix} \bar{\Pi} & \tilde{O} & \tilde{S} & \tilde{U} & \tilde{P} \\ * & -V & 0 & 0 & 0 \\ * & 0 & -P & 0 & 0 \\ * & 0 & 0 & -\epsilon_1 I & 0 \\ * & 0 & 0 & 0 & -\epsilon_2 I \end{bmatrix} < 0,$$

where

$$\bar{\Pi} = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} & \Pi_{1,3} & \Pi_{1,4} & \Pi_{1,5} & \Pi_{1,6} & \Pi_{1,7} \\ * & \Pi_{2,2} & \Pi_{2,3} & \Pi_{2,4} & \Pi_{2,5} & \Omega_{2,6} & \Pi_{2,7} \\ * & * & \Pi_{3,3} & \Pi_{3,4} & \Pi_{3,5} & \Pi_{3,6} & \Pi_{3,7} \\ * & * & * & \Pi_{4,4} & \Pi_{4,5} & \Pi_{4,6} & \Pi_{4,7} \\ * & * & * & * & \Pi_{5,5} & \Pi_{5,6} & \Pi_{5,7} \\ * & * & * & * & * & \Pi_{6,6} & \Pi_{6,7} \\ * & * & * & * & * & * & \Pi_{7,7} \end{bmatrix}$$

$\Pi_{1,1} = -PA - A^T P + R + O_1^T + O_1 + \epsilon_1 N_1^T N_1 + \epsilon_2 N_4^T N_4,\ \Pi_{1,2} = -O_1 + O_2^T + A^T \bar{U}^T + \epsilon_2 N_4^T N_5,$

$\Pi_{1,4} = PW + \epsilon_1 N_1 N_3,\ \Pi_{1,5} = PD + \epsilon_1 N_1 N_6,\ \ \Pi_{1,6} = P,\ \Pi_{1,7} = O_3^T - A^T U^T,$

$\Pi_{2,2} = -(1-d)R - O_2^T - O_2 + \epsilon_2 N_5^T N_5,\ \Pi_{2,3} = \bar{U}B + \epsilon_1 N_2 N_3,\ \Pi_{2,4} = LT_2 + \bar{U}W,\ \Pi_{2,5} = \bar{U}D,$

$\Pi_{2,6} = \bar{U},\ \Pi_{2,7} = -O_3^T - \bar{U},\ \Pi_{3,3} = Q + \bar{\tau}^2 S - 2T_1 + \epsilon_1 N_2^T N_2,\ \Pi_{3,4} = \Pi_{3,5} = \Pi_{4,5} = \Pi_{4,6} = 0,$

$\Pi_{3,6} = -I, \Pi_{3,7} = B^T\ U^T,\ \Pi_{4,7} = W^T\ U^T, \Pi_{4,4} = -(1-d)Q - 2T_2 + \epsilon_1 N_3^T N_3,\ \Pi_{5,5} = -S$

$+ \epsilon_1 N_6^T N_6,\ \Pi_{5,6} = 0,\ \Pi_{5,7} = D^T\ U^T,\ \Pi_{6,6} = -\gamma I,\ \Pi_{6,7} = U^T,\ \Pi_{7,7} = \bar{\tau}^2 V - U^T - U,$

$\Pi_{1,3} = PB + LT_1 - \epsilon_1 N_1^T N_2,\ \tilde{U}^T = [M^T P\ M^T \bar{U}^T\ 0\ 0\ 0\ 0\ M^T U^T\ 0\ 0],\ \tilde{P}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ M^T P],$

$\tilde{O}^T$ *and* $\tilde{S}^T$ *are as defined in Theorem 3.1.*

Proof: Consider the Lyapunov functional as described in Theorem 3.1. For any matrices O1, O2, O3, U, U¯ of appropriate dimensions, it can be shown that 2[x T (t − τ (t))U¯ + g T 1 (t)U][−g1(t) − (A + ∆A)x(t) + (B + ∆B)f(x(t)) + (W + ∆W)f(x(t − τ (t))) + (D + ∆D) Z t t−τ(t) f(x(s))ds + J(t) i = 0, (19) 9 By Itˆo's formula, using (12) and (19) the stochastic derivative of V (t, x(t)) along the trajectories of the system (1) can be obtained as,

dV (t, x(t)) ≤ 2x T (t)P h − (A + ∆A)x(t) + (B + ∆B)f(x(t))d + (W + ∆W)f(x(t − τ (t))) + (D + ∆D) Z t t−τ(t) f(x(s))ds + J(t) i + f T (x(t)) Q f(x(t)) + x T (t) R x(t) − (1 − d) x T (t − τ (t))Rx(t − τ (t)) + f T (x(t − τ (t))) Q f(x(t − τ (t))) + ¯τ 2 f T (x(t))Sf(x(t)) − τ¯ Z t t−τ(t) f T (x(s))Sf(x(s))ds − τ¯ Z t t−τ(t) g T 1 (s)V g1(s)ds + 2[x T (t − τ (t))U¯ + g T 1 (t)U][−g1(t) − (A + ∆A)x(t) + (B + ∆B)f(x(t)) + (W + ∆W)f(x(t − τ (t))) + (D(t) + ∆D(t)) Z t t−τ(t) f(x(s))ds + J(t) i + 2[x T (t)O1 + x T (t − τ (t))O2 + g T 1 (t)O3][x(t) − x(t − τ (t)) − Z t t−τ(t) g1(s)ds − Z t t−τ(t) g2(s)dw(s) + ¯τ 2 g T 1 (t)V g1(t) + [(C + ∆C)x(t) + (H + ∆H)x(t − τ (t))]T P[(C + ∆C)x(t) + (H + ∆H)x(t − τ (t))] + 2x T (t)P[(C + ∆C)x(t) + (H + ∆H)x(t − τ (t))]dw(t). (20)

From (15) we have,

dV (t) ≤ dV (t) − 2T1f T (x(t))[f(x(t)) − Lx(t)] − 2T2f T (x(t − τ (t)))[f(x(t − τ (t))) − Lx(t − τ (t))]. (21)

Finally, from (19)-(21), we have

dV (t, x(t)) − γJT (t)J(t) − J T (t)y(t) ≤ ξ T (t) Ξ¯ ξ(t) + ξ T 1 (t)O V −1O T ξ1(t) − 2ξ T 1 (t)O Z t t−τ(t) g2(s)dw(s) + 2x T (t)P h (C + ∆C)x(t) + (H + ∆H)x(t − τ (t))i dw(t) o , (22)

where

ξ(t) = [x T (t) x T (t − τ (t)) f T (x(t)) f T (x(t − τ (t))) Z t t−τ(t) f(x(s))dsT J T (t) g T 1 (t)]T

$$\bar{\Xi}=\begin{bmatrix} \bar{\Xi}_{1,1} & \bar{\Xi}_{1,2} & \bar{\Xi}_{1,3} & \bar{\Xi}_{1,4} & \bar{\Xi}_{1,5} & \bar{\Xi}_{1,6} & \bar{\Xi}_{1,7} \\ * & \bar{\Xi}_{2,2} & \bar{\Xi}_{2,3} & \bar{\Xi}_{2,4} & \bar{\Xi}_{2,5} & \bar{\Xi}_{2,6} & \bar{\Xi}_{2,7} \\ * & * & \bar{\Xi}_{3,3} & \bar{\Xi}_{3,4} & \bar{\Xi}_{3,5} & \bar{\Xi}_{3,6} & \bar{\Xi}_{3,7} \\ * & * & * & \bar{\Xi}_{4,4} & \bar{\Xi}_{4,5} & \bar{\Xi}_{4,6} & \bar{\Xi}_{4,7} \\ * & * & * & * & \bar{\Xi}_{5,5} & \bar{\Xi}_{5,6} & \bar{\Xi}_{5,7} \\ * & * & * & * & * & \bar{\Xi}_{6,6} & \bar{\Xi}_{6,7} \\ * & * & * & * & * & * & \bar{\Xi}_{7,7} \end{bmatrix},$$

$$\bar{\Xi}_{1,1}=-P(A+\Delta A)-(A+\Delta A)^TP+R+(C+\Delta C)^TP(C+\Delta C)+O_1^T+O_1,$$
$$\bar{\Xi}_{1,2}=(C+\Delta C)^TP(H+\Delta H)-(A+\Delta A)^T\bar{U}^T-O_1+O_2^T,\ \bar{\Xi}_{1,3}=P(B+\Delta B)+LT_1,$$
$$\bar{\Xi}_{1,4}=P(W+\Delta W),\ \bar{\Xi}_{1,5}=P(D+\Delta D)0,\ \ \bar{\Xi}_{1,6}=P,\ \ \bar{\Xi}_{1,7}=O_3^T-(A+\Delta A)^TU^T,$$
$$\bar{\Xi}_{2,2}=(H+\Delta H)^TP(H+\Delta H)-(1-d)R-O_2^T-O_2,\ \bar{\Xi}_{2,3}=\bar{U}(B+\Delta B),$$
$$\bar{\Xi}_{2,4}=LT_2+\bar{U}(W+\Delta W),\ \bar{\Xi}_{2,5}=\bar{U}(D+\Delta D),\ \bar{\Xi}_{2,6}=\bar{U},\ \bar{\Xi}_{2,7}=-O_3^T-\bar{U},\ \bar{\Xi}_{3,3}=Q+\bar{\tau}^2S-2T_1,$$
$$\bar{\Xi}_{3,4}=0,\ \bar{\Xi}_{3,5}=0,\ \bar{\Xi}_{3,6}=-I,\ \bar{\Xi}_{3,7}=(B+\Delta B)^TU^T,\ \bar{\Xi}_{4,4}=-(1-d)Q-2T_2,$$
$$\bar{\Xi}_{4,5}=0,\ \bar{\Xi}_{4,6}=0,\ \bar{\Xi}_{4,7}=(W+\Delta W)U^T,\bar{\Xi}_{5,5}=-S,\ \bar{\Xi}_{5,6}=0,$$
$$\bar{\Xi}_{5,7}=(D+\Delta D)U^T,\ \bar{\Xi}_{6,6}=-\gamma I,\ \bar{\Xi}_{6,7}=U^T,\ \bar{\Xi}_{7,7}=\bar{\tau}^2V-U-U^T.$$

By using Schur complement Lemma 2.4, we have

$$\bar{\Xi}=\begin{bmatrix} \Sigma_{1,1} & \Sigma_{1,2} & \Sigma_{1,3} & \Sigma_{1,4} & \Sigma_{1,5} & \Omega_{1,6} & \Sigma_{1,7} & \Sigma_{1,8} \\ * & \Omega_{2,2} & \Omega_{2,3} & \Sigma_{2,4} & \Sigma_{2,5} & \Omega_{2,6} & \Omega_{2,7} & \Sigma_{2,8} \\ * & * & \Omega_{3,3} & \Omega_{3,4} & \Omega_{3,5} & \Omega_{3,6} & \Sigma_{3,7} & 0 \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & \Omega_{4,6} & \Sigma_{4,7} & 0 \\ * & * & * & * & \Omega_{5,5} & \Omega_{5,6} & \Sigma_{5,7} & 0 \\ * & * & * & * & * & \Omega_{6,6} & \Omega_{6,7} & 0 \\ * & * & * & * & * & * & \Omega_{7,7} & 0 \\ * & * & * & * & * & * & * & \Sigma_{8,8} \end{bmatrix},$$

where

$$\Sigma_{1,1}=\Omega_{1,1}-P\Delta A-\Delta A^TP,\ \Sigma_{1,2}=\Omega_{1,2}-\Delta A^T\bar{U}^T,\ \Sigma_{1,3}=\Omega_{1,3}+P\Delta B,$$
$$\Sigma_{1,4}=\Omega_{1,4}+P\Delta W,\ \Sigma_{1,5}=\Omega_{1,5}+P\Delta D,\Sigma_{1,7}=\Omega_{1,7}-\Delta A^TU^T,\ \Sigma_{1,9}=(C+\Delta C)^TP,$$
$$\Sigma_{2,3}=\Omega_{2,3}+\bar{U}\Delta B(t),\ \Sigma_{2,4}=\Omega_{2,4}+\bar{U}\Delta W(t),\ \Sigma_{2,8}=(H+\Delta H)^TP,$$
$$\Sigma_{3,7}=\Omega_{3,7}+\Delta B^TU^T,\ \Sigma_{4,7}=\Omega_{4,7}+\Delta W^T(t)U^T,,\ \Sigma_{5,7}=\Omega_{5,7}+\Delta D^T(t)U^T,\ \Sigma_{8,8}=-P.$$

$\bar{\Xi}$ can be written as

$$\bar{\Xi} = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & \Omega_{1,3} & \Omega_{1,4} & \Omega_{1,5} & \Omega_{1,6} & \Omega_{1,7} & C^T P \\ * & \Omega_{2,2} & \Omega_{2,3} & \Omega_{2,4} & \Omega_{2,5} & \Omega_{2,6} & \Omega_{2,7} & H^T P \\ * & * & \Omega_{3,3} & \Omega_{3,4} & \Omega_{3,5} & \Omega_{3,6} & \Omega_{3,7} & 0 \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & \Omega_{4,6} & \Omega_{4,7} & 0 \\ * & * & * & * & \Omega_{5,5} & \Omega_{5,6} & \Omega_{5,7} & 0 \\ * & * & * & * & * & \Omega_{6,6} & \Omega_{6,7} & 0 \\ * & * & * & * & * & * & \Omega_{7,7} & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix}$$
$$+ \Lambda_1 F(t)\Lambda_2^T + \Lambda_2 F^T(t)\Lambda_1 + \Lambda_3 F(t)\Lambda_4^T + \Lambda_4 F^T(t)\Lambda_3$$

where

$$\Lambda_1 = \begin{bmatrix} PM \\ \bar{U}M \\ 0 \\ 0 \\ 0 \\ 0 \\ UM \\ 0 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} -N_1^T \\ 0 \\ N_2^T \\ N_3^T \\ N_6^T \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Lambda_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ PM \end{bmatrix}, \quad \Lambda_4 = \begin{bmatrix} N_4^T \\ N_5^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

By using Lemma 2.5 we have

$$\bar{\Xi} \le \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & \Omega_{1,3} & \Omega_{1,4} & \Omega_{1,5} & \Omega_{1,6} & \Omega_{1,7} & C^T P \\ * & \Omega_{2,2} & \Omega_{2,3} & \Omega_{2,4} & \Omega_{2,5} & \Omega_{2,6} & \Omega_{2,7} & H^T P \\ * & * & \Omega_{3,3} & \Omega_{3,4} & \Omega_{3,5} & \Omega_{3,6} & \Omega_{3,7} & 0 \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & \Omega_{4,6} & \Omega_{4,7} & 0 \\ * & * & * & * & \Omega_{5,5} & \Omega_{5,6} & \Omega_{5,7} & 0 \\ * & * & * & * & * & \Omega_{6,6} & \Omega_{6,7} & 0 \\ * & * & * & * & * & * & \Omega_{7,7} & 0 \\ * & * & * & * & * & * & * & -P \end{bmatrix}$$
$$+ \epsilon_1 \Lambda_1^T \Lambda_1 + \epsilon_1^{-1} \Lambda_2^T \Lambda_2 + \epsilon_2 \Lambda_3^T \Lambda_3 + \epsilon_2^{-1} \Lambda_4^T \Lambda_4.$$

Again by using Schur complement lemma 2.4 Ξ can be written as Π. Taking the mathematical ¯ expectation on the both sides of (22), one can deduce that E{dV (t, x(t)) − γJT (t)J(t) − J T (t)u(t)} ≤ E(ξ T (t)Πξ(t)) ≤ 0, which means E{ Z t 0 J T (s)y(s)ds} ≥ E V (t, x(t)) − V (t, 0) − γ Z t 0 J T (s)J(s)ds 12 = E V (t, x(t)) − γ Z t 0 J T (s)J(s)ds ≥ −γE Z t 0 J T (s)J(s)ds . From Definition 2.2, this indicates that the stochastic neural network (4) is globally robustly passive in the sense of expectation, and the proof of Theorem 4.1 is completed.

If there are no distributed delays then the system (1) is simplified to dx(t) = [−(A + ∆A)x(t) + (B + ∆B)f(x(t) + (W + ∆W)f(x(t − τ (t))) + J(t)]dt + [(C + ∆C)x(t) + (H + ∆H)x(t − τ (t))]dw(t), (23) Corollary 4.2. The delayed neural networks (23) is said to be globally passive in the sense of expectation, if there exist symmetric positive definite matrices P > 0, Q > 0, R > 0, S > 0, V > 0, positive diagonal matrices T1 > 0, T2 > 0 any matrices O1, O2, O3, U, U¯ and positive scalars 1, 2 such that feasible solution exist for the following LMI,

$$\Pi = \begin{bmatrix} \bar{\Pi}_1 & \tilde{O} & \tilde{S} & \tilde{U} & \tilde{P} \\ * & -V & 0 & 0 & 0 \\ * & 0 & -P & 0 & 0 \\ * & 0 & 0 & -\epsilon_1 I & 0 \\ * & 0 & 0 & 0 & -\epsilon_2 I \end{bmatrix} < 0, \qquad (24)$$

where

$$\bar{\Pi} = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} & \Pi_{1,3} & \Pi_{1,4} & \Pi_{1,5} & \Pi_{1,6} & \Pi_{1,7} \\ * & \Pi_{2,2} & \Pi_{2,3} & \Pi_{2,4} & \Pi_{2,5} & \Omega_{2,6} & \Pi_{2,7} \\ * & * & \Pi_{3,3} & \Pi_{3,4} & \Pi_{3,5} & \Pi_{3,6} & \Pi_{3,7} \\ * & * & * & \Pi_{4,4} & \Pi_{4,5} & \Pi_{4,6} & \Pi_{4,7} \\ * & * & * & * & \Pi_{5,5} & \Pi_{5,6} & \Pi_{5,7} \\ * & * & * & * & * & \Pi_{6,6} & \Pi_{6,7} \\ * & * & * & * & * & * & \Pi_{7,7} \end{bmatrix}$$

$$\Pi_{1,1} = -PA - A^TP + R + O_1^T + O_1 + \epsilon_1 N_1^T N_1 + \epsilon_2 N_4^T N_4,\ \Pi_{1,2} = -O_1 + O_2^T$$
$$+ A^T \bar{U}^T + \epsilon_2 N_4^T N_5,\ \Pi_{1,4} = PW + \epsilon_1 N_1 N_3,\ \Pi_{1,5} = 0,\ \ \Pi_{1,6} = P,\ \Pi_{1,7} = O_3^T - A^T U^T,$$
$$\Pi_{2,2} = -(1-d)R - O_2^T - O_2 + \epsilon_2 N_5^T N_5,\ \Pi_{2,3} = \bar{U}B + \epsilon N_2 N_3,\ \Pi_{2,4} = LT_2 + \bar{U}W,\ \Pi_{2,5} = 0,$$
$$\Pi_{2,6} = \bar{U},\ \Pi_{2,7} = -O_3^T - \underline{\bar{U}},\ \Pi_{3,3} = Q + \bar{\tau}^2 S - 2T_1 + \epsilon_1 N_2^T N_2,\ \ \Pi_{3,4} = \Pi_{3,5} = \Pi_{4,5} = \Pi_{4,6} = 0,$$
$$\Pi_{3,6} = -I, \Pi_{3,7} = B^T\ U^T,\ \Pi_{4,7} = W^T\ U^T, \Pi_{4,4} = -(1-d)Q - 2T_2 + \epsilon_1 N_3^T N_3,\ \Pi_{5,5} = -S,$$
$$\Pi_{5,6} = 0,\ \Pi_{5,7} = 0,\ \Pi_{6,6} = -\gamma I,\ \Pi_{6,7} = U^T,\ \Pi_{7,7} = \bar{\tau}^2 V - U^T - U,\ \Pi_{1,3} = PB + LT - \epsilon_1 N_1^T N_2,$$
$$\tilde{U}^T = [M^T P\ M^T \bar{U}^T\ 0\ 0\ 0\ 0\ M^T U^T\ 0\ 0],\ \tilde{P}^T = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ M^T P],$$

O˜T and S˜T are as def ined in Theorem 3.1.

Proof: The proof is similar to that in the proof of Theorem 4.1 by choosing D=0. Hence it is omitted. If there are no stochastic disturbance then the system (23) is simplified to dx(t) = −(A + ∆A)x(t) + (B + ∆B)f(x(t)) + (W + ∆W)f(x(t − τ (t))) + J(t), (25) y(t) = f(x(t)), x(t) = φ(t), −τ¯ ≤ t ≤ 0. Corollary 4.3. The delayed neural networks (25) is said to be globally passive, if there exist symmetric positive definite matrices P > 0, Q > 0, R > 0, S > 0, V > 0, positive diagonal matrices T1 > 0, T2 > 0 any matrices O1, O2, O3, U, U¯ and positive scalar 1 such that feasible solution exist for the following LMI,

$$\tilde{\Omega} = \begin{bmatrix} \Omega_{1,1} & \Omega_{1,2} & \Omega_{1,3} & \Omega_{1,4} & \Omega_{1,5} & \Omega_{1,6} & \Omega_{1,7} \\ * & \Omega_{2,2} & \Omega_{2,3} & \Omega_{2,4} & \Omega_{2,5} & \Omega_{2,6} & \Omega_{2,7} \\ * & * & \Omega_{3,3} & \Omega_{3,4} & \Omega_{3,5} & \Omega_{3,6} & \Omega_{3,7} \\ * & * & * & \Omega_{4,4} & \Omega_{4,5} & \Omega_{4,6} & \Omega_{4,7} \\ * & * & * & * & \Omega_{5,5} & \Omega_{5,6} & \Omega_{5,7} \\ * & * & * & * & * & \Omega_{6,6} & \Omega_{6,7} \\ * & * & * & * & * & * & \Omega_{7,7} \end{bmatrix}$$

$\Omega_{1,1} = -PA - A^T P + R + O_1^T + O_1 + \epsilon_1 N_1^T N_1,\ \Omega_{1,2} = -O_1 + O_2^T + A^T W^T,$

$\Omega_{1,3} = PB + LT_1 - \epsilon_1 N_1^T N_2,\ \Omega_{1,4} = PW + \epsilon_1 N_1 N_3,\ \Omega_{1,5} = 0,\ \ \Omega_{1,6} = P,\ \Omega_{1,7} = O_3 - A^T U^T,$

$\Omega_{2,2} = -(1-d)R - O_2^T - O_2,\ \Omega_{2,3} = WB + \epsilon_1 N_2 N_3,\ \Omega_{2,4} = LT_2 + WC,\ \Omega_{2,5} = 0,$

$\Omega_{2,6} = W,\ \Omega_{2,7} = -O_3^T - W - W^T,\ \Omega_{3,3} = Q + \bar{\tau}^2 S - 2T_1 + \epsilon_1 N_2^T N_2,\ \Omega_{3,4} = \Omega_{3,5} = \Omega_{4,5} = \Omega_{4,6} = 0,$

$\Omega_{3,6} = -I,\ \Omega_{3,7} = B^T\ U^T,\ \Omega_{4,7} = W^T\ U^T, \Omega_{4,4} = -(1-d)Q - 2T_2 + \epsilon_1 N_3^T N_3,\ \Omega_{5,5} = -S,$

$\Omega_{5,6} = 0,\ \Omega_{5,7} = 0,\ \Omega_{6,6} = -\gamma I,\ \Omega_{6,7} = U^T,\ \Omega_{7,7} = \bar{\tau}^2 V - U^T - U.$

Remark 4.4. Theorem 3.1, Theorem 4.1 and corollary 4.2 are depends on the size of time delay element, which means that it is less conservative than those delay-independent ones, especially when the delay value is very small, and this may create a gap of LMI-based delay-dependent results for stochastic delayed neural networks. It is realized that, the problems studied in many other papers are also a special class of this paper, such as [5], [29], [31], and [34].

5. Numerical Examples Example 1. Consider the stochastic recurrent neural network (1) with uncertainties is of the following form : dx(t) = [−(A + ΔA)x(t) + (B + ΔB)f(x(t)) + (W + ΔW)f(x(t − τ (t))) + (D + ΔD) Z t t−τ(t) f(x(s))ds + J(t)]dt + [(C + ΔC)x(t) + (H + ΔH)x(t − τ (t))]dw(t), with the parameters A = " 3 0 0 4 # , B = " 0.1 0.9 0.8 1.2 # , W = " 0.2 0.4 0.3 0.5 # , D = " 0.1 0.1 0.3 0.3 # , C = " 0.3 0 0 0.5 # , H = " 0.1 0 0 0.2 # , L = 0.5I, N1 = N2 = N3 = 0.2I, N4 = N5 = N6 = 0.1I, M = 0.2I. By using the Matlab LMI toolbox [12], we solve the LMI (18) for i > 0 (i = 1, 2), τ = 0.9 and d = 1.5 the feasible solutions are P = " 35.9002 9.6420 9.6420 35.3566 # , Q = " 0.0399 −0.0263 −0.0263 0.0179 # , R = " 0.0025 −0.0015 −0.0015 0.0011 # , S = " 31.3169 29.4440 29.4440 30.7951 # , V = " 8.7753 0.5379 0.5379 7.0063 # , T1 = " 339.8946 0 0 383.0669 # , T2 = " 11.9886 0 0 13.5799 # , γ = 9.7467.

Therefore, the concerned neural networks with time-varying delays is passive.

Example 2. Consider the recurrent neural network [11] is of the following form:

x˙(t) = −(A + ∆A(t))x(t) + (B + ∆B)f(x(t)) + (W + ∆W(t))f(x(t − τ (t))) + J(t)

with the parameters A = " 2 0 0 3.5 # , B = " −0.34 −0.44 0.38 −0.03 # , W = " 0.22 1.35 1.51 −0.41 # , L = " 0.3 0 0 0.4 # , M = " 0.1 −0.2 0.2 0.3 # , N1 = [0.02 0.02], N2 = [−0.03 0.01], N3 = [0.01 − 0.01]

By using the Matlab LMI toolbox [12], we solve the LMI (26) for 1 > 0, τ¯ = 0.8629 and d = 0.9 the feasible solutions are 15 P = " 47.1846 −157.1489 −157.1489 850.9588 # , Q = " 0.0007 −0.0012 −0.0012 0.0276 # , R = " 0.0014 −0.0068 −0.0068 0.0559 # , S = " 0.0095 −0.0284 −0.0284 0.2352 # , V = " 47.9259 −159.5879 −159.5879 907.6666 # , T1 = " 44.4165 0 0 147.2487 # , T2 = " 20.3622 0 0 95.9050 # , γ = 1.7782 × 108.

Therefore, the concerned neural networks with time-varying delays is passive. Some comparisons on upper bounds of time delay are listed in Table 1. From Table 1, it can be concluded that the obtained results in this paper are less conservative than those in [9, 11, 24]. 6. Conclusion In this paper, we have considered the problem of passivity and robust passivity analysis for a class of uncertain stochastic recurrent neural networks with time-varying delays. By choosing a new Lyapunov-Krasovskii functional, the improved delay-dependent criteria have been proposed. Finally, numerical examples have been provided to illustrate the effectiveness of the obtained results. Our results can be specialized to several cases including those studied extensively in the literature. Table 1. Comparision of the maximal allowable delays ¯τ for Example 2 Method d=0.3 d=0.5 d=0.7 d=0.9 [24] 0.1178 0.1145 0.1123 0.1105 [9] 0.4197 0.4145 0.4117 0.4082 [11] 0.5763 0.5679 0.5566 0.5273 Corollary 4.2 9.2211 1.4397 0.9149 0.8629

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