

Multiclass Fault Diagnosis of Rolling Element Bearing Based on Empirical Mode Decomposition and Entropy Features

Swarup K Laha*

Condition Monitoring Group, CSIR-Central Mechanical Engineering Research Institute, West Bengal, India

Abstract

Rolling element bearings are one of the most commonly used machine elements in engineering industry. Fault detection and diagnosis of rolling element bearings is essential for prevention of malfunction and failure during operation. The present work deals with rolling element bearing fault diagnosis by using Support Vector Machine (SVM) and Artificial Neural Network (ANN). Four types of bearing conditions are considered in the present analysis: inner race fault (IR), outer race fault (OR), ball fault (BF) and healthy bearing (HB). The vibration signals from bearing housing are acquired through accelerometers. Since the vibration signal from faulty bearing is non-stationary and non-linear Empirical Mode Decomposition (EMD) is a suitable method for analyzing such signals. The vibration signal is decomposed into intrinsic mode functions (IMF) by using EMD method. Statistical features like Shannon Entropy and Approximate Entropy from the IMFs are extracted for training and testing of the SVM and ANN. The trained models are able to classify different kinds of faults with good accuracy.

Keywords: ANN, approximate entropy, EMD, rolling-element bearing fault, Shannon entropy, spectral entropy, SVM

*Corresponding Author

E-mail: swarup.laha@gmail.com

INTRODUCTION

Rolling element bearings are extensively used in vast majority of rotating equipment. Rolling element bearings consist of inner race, outer race, cage and ball. Due to excessive wear or damage in these parts the bearing exhibits high vibration and may fail during operation. Therefore, condition monitoring based on vibration analysis of bearings is essential for fault diagnosis and prevention of malfunction of machines.

Condition monitoring of faulty rolling element bearing using vibration signal has been carried out by many authors. The vibration signal used for fault diagnosis mainly falls into three categories (i) time domain signal; ^[1-3, 8] (ii) frequency domain signal ^[3, 10, 20] and (iii) time-frequency

domain signal. ^[11-13] Then, many pattern recognition techniques such as ANN ^[1-2, 6, 12-13], SVM, ^[12-13] ANFIS, ^[3, 15] HMM ^[19] etc. had been used for fault diagnosis of rolling element bearing.

Samanta and Al-Balushi ^[1] used ANN for fault diagnosis of rolling element bearing by using statistical features extracted directly from time-domain vibration signal segments. Subrahmanyam and Sujatha ^[2] used neural networks for the diagnosis of localized defects in ball bearings. Lei et al. ^[3] proposed a new fault diagnosis method based on an improved distance evaluation technique and adaptive neuro-fuzzy inference system (ANFIS). They used time-domain, frequency-domain and empirical mode decomposition (EMD)

energy entropies as fault features and using the proposed improved distance evaluation technique the most superior features are selected. Altmann and Mathew^[4] presented a novel method for detection and diagnosis of slow speed rolling element bearing using discrete wavelet packet analysis combined with multiple band-pass filtered signal with autoregressive (AR) spectrum of the envelop signal. Hui-Li, Zhang and Zheng^[5] presented an novel approach for fault diagnosis of rolling element bearing based on order tracking, empirical mode decomposition (EMD) and Teager Kaiser energy operator (TKEO) technique. Yang et al.^[6] used neural network and EMD to extract the energy of different frequency bands as features to identify roller bearing fault patterns accurately and effectively. They showed that the combination of EMD and ANN provides a better fault diagnosis method for normal, inner race and outer race faults than the combination of ANN and wavelet technique. Dong et al.^[7] presented an improved shifting process for EMD method resulting in shortened time while maintaining the same accuracy. The improved EMD process combined with Shock Pulse Method (SPM) and demodulation was efficient and accurate and could be effectively applied roller bearing fault diagnosis. Heng and Nor^[8] studied sound pressure and vibration signal of rolling element bearing using various statistical parameters like skewness, crest factor, kurtosis etc. Rai and Mohanty^[9] applied EMD process to extract IMFs and then carried out FFT analysis of IMFs to identify the characteristic bearing defect frequencies. Taylor^[10] identified defects in antifriction bearings from analyses of the low frequencies (up to 2,000 Hz) generated by the moving parts in the bearing by using spectral method. Jena and Panigrahi^[11] applied Continuous Wavelet Transform (CWT) to study various fault features of rolling element bearing. Kankar et al.^[12] studied rolling element bearing

fault diagnosis using wavelet transform. Seven different base wavelets were considered for the study and Complex Morlet wavelet was selected as it had minimum Shannon Entropy for extracting statistical features. Then three different classification algorithms namely Support Vector Machine (SVM), Learning Vector Quantization (LVQ) and Self-Organizing Map (SOM) were used for bearing fault classification. Kankar et al.^[13] also studied fault features of rolling bearing using cyclic autocorrelation of raw vibration signals.

Three learning techniques were used for faults classifications: Support vector machine, Artificial Neural Network and Self-Organizing Maps. They showed that SVM gave better and accurate classification performance. Liu et al.^[14] applied correlation matching as feature extraction algorithm for automatic bearing fault diagnosis. Lou and Laparo^[15] used wavelet analysis combined with adaptive neural-fuzzy inference system (ANFIS) for diagnosing localized defects in rolling element bearing. Mcfadden and Toozhy^[16] used high frequency resonance technique (HFRT) with synchronous averaging for studying rolling element bearings with spalling damage. Wang et al.^[17] used Autoregressive (AR)/Autoregressive Conditional Heteroscedasticity (ARCH) model coefficients from the bearing vibration signal for extracting the feature vectors.

Then they used K-means based clustering method for automatic classification of bearing fault status. Zhang et al.^[18] presented a bearing fault diagnosis method based on multi-scale entropy (MSE) as the vibration signal is nonlinear characteristics and then used adaptive neuro-fuzzy inference system (ANFIS) for fault classification. The proposed approach was not only very reliable and efficient in fault classification, but identifies the level of fault severity accurately. Purushotham et

al. ^[19] presented a bearing fault diagnosis system using discrete wavelet analysis (DWT) and Hidden Markov Model (HMM).

Swahili et al. ^[20] used Spectral Kurtosis (SK) method for bearing diagnostics. Further, AR-based linear prediction filter combined with Minimum Entropy Deconvolution (MED) was used for bearing signal enhancement.

In the present analysis bearings with healthy condition (HB) and with inner race (IR), outer race (OR) and ball fault (BF) are taken up for experimental simulation. The acceleration vibration signal from the bearing is acquired through a DAQ system.

The vibration signal is decomposed into IMFs by the EMD process. A brief description of the EMD process is given in section 2.

The fault features from the IMFs are extracted from three entropy definitions: Shannon entropy, approximate entropy and spectral entropy. Section 3 provides a brief description of these entropies. Two soft computing techniques, ANN and SVM are employed for diagnosis of bearing faults using the extracted features.

EMPIRICAL MODE DECOMPOSITION

Empirical Mode Decomposition (EMD) is a time domain signal processing method in which the signal is decomposed into a number of simpler signals called Intrinsic Mode Functions (IMF).

The IMF signals have the following properties: (a) only one extreme between zero crossings and (b) a mean value of zero. The process is useful for analyzing non-linear and non-stationary signals. The process was proposed by Huang et al. ^[21-22].

The procedure of extracting an IMF is called sifting. The sifting procedure is as follows:

1. For a signal $X(t)$, let m_1 be the mean of its upper and lower envelopes as determined from a cubic-spline interpolation of local maxima and minima.

2. The first component h_1 is computed as:

$$h_1 = X(t) - m_1 \quad (1)$$

3. In the second sifting process, h_1 is treated as the data, and m_{11} is the mean of h_1 's upper and lower envelopes, thus:

$$h_{11} = h_1 - m_{11} \quad (2)$$

4. This sifting procedure is repeated k times, until h_{1k} is an IMF, that is:

$$h_{1(k-1)} - m_{1k} = h_{1k} \quad (3)$$

5. Then it is designated as $c_1 = h_{1k}$, the first IMF component from the signal, which contains the shortest period component of the signal. It is separated from the rest of the data by subtracting from the original data:

$$X(t) - c_1 = r_1 \quad (4)$$

Since the residue, r_1 , still contains longer period variations in the data, it is treated as the new data and subjected to the same sifting process as described above. The procedure is repeated on successive r_j 's as follows:

$$r_1 - c_2 = r_2, \dots, r_{n-1} - c_n = r_n \quad (5)$$

6. The sifting process stops finally when the residue, r_n , becomes a monotonic function from which no more IMF can be extracted. From the above equations, we can reconstruct the original signal as such,

$$X(t) = \sum_{j=1}^n c_j + r_n \quad (6)$$

Thus following the above shifting procedure a number of IMFs and a residue, r_n can be obtained.

3. Feature Extraction:

Feature extraction is one of the most important steps for classification algorithm.

Following the above discussion a number of IMF signals can be obtained from the vibration signal. From each IMF a number of statistical features can be derived. In the present work the following statistical

features are extracted to detect incipient bearing damage:

3.1 Shannon Entropy:

Shannon entropy is an important statistical property of a signal which gives the information content of the signal. The entropy can explicitly be written as,

$$E = -\sum_{i=1}^N p(X_i) \log_2 p(X_i) \quad (7)$$

Where $X = \{X_1, X_2, \dots, X_n\}$ is a random phenomenon and $p(X_i)$ is the probability of X_i .

3.2 Approximate Entropy:

Approximate entropy (*ApEn*) is a measurement technique for regularity statistic and unpredictability of fluctuations over a time-series data. The complexity measure was developed by Pincus^[23]. For an N sample raw time series data equi-spaced in time, $\{u(i): 1 \leq i \leq N\}$, a sequence of vector $X(1), X(2), \dots, X(N-m+1)$ in $\mathfrak{R}^m$ can be formed as follows:

$$X_i^m = \{u(i), u(i+1), \dots, u(i+m-1)\}, i=1, \dots, N-m+1 \quad (8)$$

Where, m is an integer giving the length of the compared window.

The above sequence $X(1), X(2), \dots, X(N-m+1)$ is used to construct for each i , $1 \leq i \leq N-m+1$

$$C_i^m(r) = \text{number of } X(j) \text{ such that } d[X(i), X(j)] \leq r / (N-m+1) \quad (9)$$

in which $d[X(i), X(j)]$ is defined as

$$d[X(i), X(j)] = \max_{k=1, 2, \dots, m} (|u(i+k-1) - u(j+k-1)|) \quad (10)$$

$$\Phi^m(r) = (N-m+1)^{-1} \sum_{i=1}^{N-m+1} \log C_i^m(r) \quad (11)$$

Further, let us define

In the above definition r defines the level of filtering. Finally, the approximate entropy is given as,

$$ApEn(m, r) = \Phi^{m+1}(r) - \Phi^m(r). \quad (12)$$

A low value of *ApEn* indicates that the data is regular and predictable.

SPECTRAL ENTROPY

Entropy is a measures the impulsiveness of a signal or a probability mass function (PMF). Thus, a PMF with sharp peak will

have low entropy while a PMF with flat distribution will have high entropy. The PMF of a signal spectrum is given by,

$$x_i = \frac{X_i}{\sum_{i=1}^N X_i} \text{ For } i=1 \text{ to } N. \quad (13)$$

Where, X_i is the energy of i^{th} frequency component of the spectrum obtained from the power spectral density of an IMF signal; thus, $\mathbf{x} = (x_1, x_2, \dots, x_N)$ is the PMF of the spectrum and N is the number of

points in the spectrum. Finally, the spectral entropy from the vector $\mathbf{X}$ is calculated as

$$E = -\sum_{i=1}^N x_i \log_2 x_i \quad (14)$$

For classification, different sets of feature vectors are obtained from IMFs and their statistical measures as defined above.

CLASSIFICATION ALGORITHM

Classification in machine learning is the process of grouping a new set of observation data, based on training data set whose category membership is known. In machine learning, classification is of two types: supervised learning, where training set of correctly identified observations is available *a-priori* and unsupervised learning or cluster analysis, where grouping of data is based on some inherent statistical measures. In the present analysis, two Support Vector Machine (SVM) and Artificial Neural Network (ANN), which are supervised learning methods, are adopted for rolling element bearing fault diagnosis. The mechanism of SVM and ANN are widely available in literature, so they are not reproduced here. The Radial Basis Function (RBF) is used in the present analysis as the kernel function for SVM with 10-fold cross validation. For training a maximum iteration number of 1000 and mean square error (MSE) of 0.001 are used. Similarly, the ANN classification is based on Levenberg-Marquardt backpropagation algorithm. For training, a target mean

square error (MSE) of $1e-10$, a minimum gradient of $1e-15$ and maximum iteration number (epoch) of 500 are used.

EXPERIMENTAL SET-UP

In the present work Machinery Fault Simulator (MFS) test rig from SpectraQuest Inc. is used for vibration analysis of the healthy and faulty rolling element bearing. The rotor-bearing system is connected with a Variable Frequency Drive (VFD) motor. The rotor-shaft system is supported on two rolling element bearings. ER12K series of standard duty bearings from Rexnord are used in the experiment. Four types of bearing fault conditions are introduced in the bearing: inner race fault (IR), outer race fault (OR), ball fault (BF) and healthy bearing (HB). The faulty bearings are mounted on non-drive end of the rotor-bearing system as shown in the figure. The test NI Compact DAQ hardware system is used for data acquisition. NI LabVIEW developer suit is used for vibration data acquisition, storage and analysis.

Piezo-electric accelerometers (IMI 608A11) are used for picking up the vibration signals from bearing housing. Sensitivity of these accelerometers is 100 mV/g and the measurement range is $\pm 50g$. Optical tachometer is used for measuring rotor speed. A schematic diagram of the test rig is shown in the figure below.

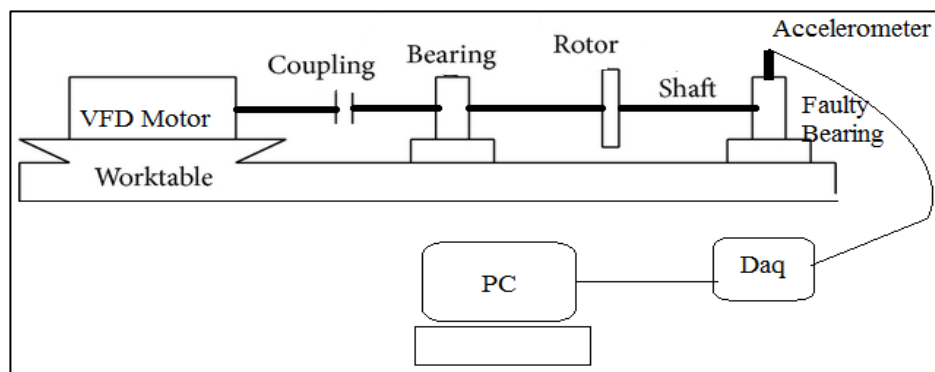


Fig. 1: Schematic diagram of the test rig

The parameters of the experimental system are as follows:

Table 1: Rotor-Bearing Parameters

Length of shaft	660 mm
Diameter of shaft	19.05mm
Mass of Disk	653 gm
Shaft speed	20, 30, 40 Hz
No of balls	8
Ball Diameter	7.937 mm
Inner race radius	9.525mm
Outer race radius	20.707mm
Bearing Clearance	13 μm

METHODOLOGY

In the present work healthy as well as faulty rolling element bearings are mounted on the test rig which is run by a VFD motor. The rotor is run at a particular frequency. Acceleration signal from the bearing housing is acquired with a sampling frequency of 5120Hz for a period of 4 seconds. Therefore, there are 20480 data points in the entire signal length. The acceleration signal is then partitioned into 20 sections, each consisting of 1024 data points. Two sets of data are collected, thus giving total 40 sections. Empirical mode decomposition (EMD) is then applied on each sectioned signal to obtain their IMFs. Only first seven IMFs are considered for feature

extraction in the present work. Various statistical features are then extracted from the IMFs as discussed in section 3. The above procedure is then repeated for all four bearing conditions namely, inner race fault (IR), outer race fault (OR), ball fault (BF) and healthy bearing (HB). Then, the size of entire feature matrix for all four types of bearing is $160 \times 7.70\%$ of the data is used for training and the remaining 30% for testing the classification algorithm. Vibration signal from the test rig are acquired at three rotor speeds: 20Hz, 30Hz and 40Hz. Performance analysis of ANN and SVM is carried out at all three rotor speeds.

RESULT AND DISCUSSION

The vibration acceleration signal along the horizontal and vertical direction are acquired through accelerometers under healthy and with inner race, outer race and ball fault condition. The vertical vibration responses under these conditions are shown in Fig. 2. Empirical mode decomposition (EMD) is carried out on the raw signal to extract a number of IMFs. As a sample example, first four IMFs of the vibration response under ball fault condition are shown in Fig. 3.

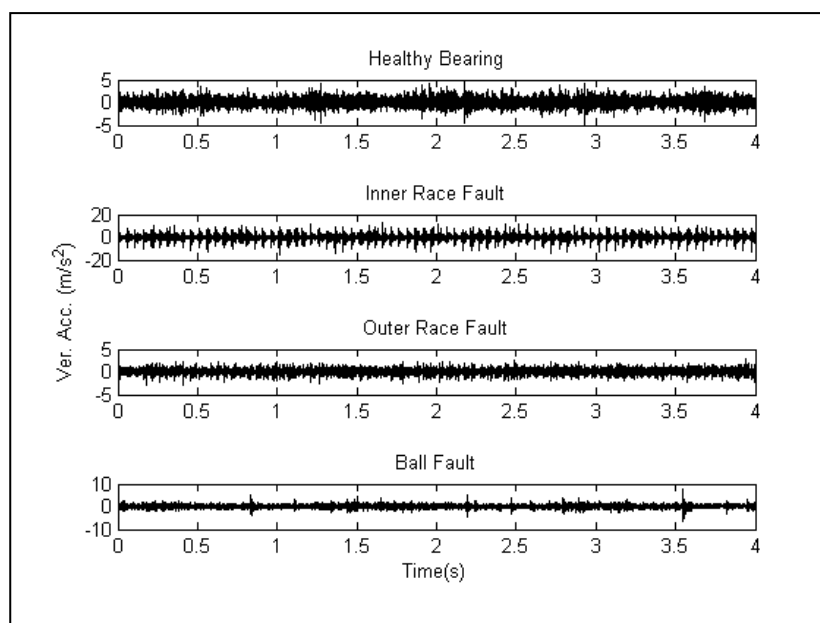


Fig. 2: Acceleration signal from bearing housing under healthy and faulty conditions

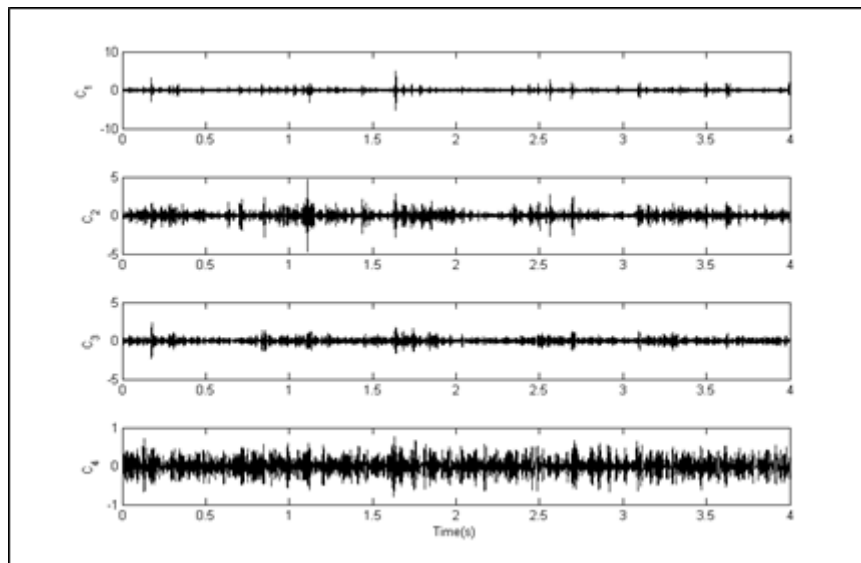


Fig. 3: First four IMFs of vertical vibration signal under ball fault

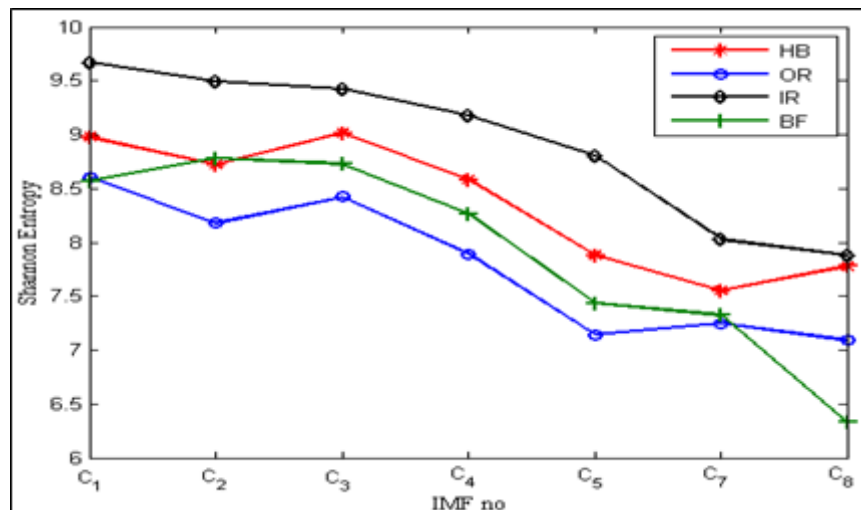


Fig. 4: Shannon Entropy vs. IMF no.

Feature vectors for training and testing the ANN and SVM as given in the section 3.0 are extracted from each IMF. Figure 4 shows the Shannon entropy vs. IMF number for different bearing fault condition. It may be deduced from Fig. 4 that entropy value reduces as the IMF number increases. Thus, higher IMFs contain lesser information.

In the present analysis, multi-class pattern recognition of bearing faults is carried out by Artificial Neural Network (ANN) and Support Vector Machine (SVM) using the features as described above. The performances of ANN in terms of

classification accuracy and number of epochs (iterations) for different statistical features are listed in Table 2.

It may be seen from Table 2 that for ANN very good training and testing classification accuracy is obtained when Shannon entropy and approximate entropy are used as classification feature for all three speeds. Classification accuracies for Shannon entropy are 97.5%, 86.9% and 93.8% when the rotor speeds are 20Hz, 30Hz and 40Hz respectively. When approximate entropy is used as the feature vector the classification accuracies are 96.9%, 85.6% and 96.3% respectively.

However, spectral entropy as feature vector gives slightly less classification accuracies which are 86.3%, 73.3% and 91.3% respectively. The epoch at which the neural network converges are also given in the Table 2.

Similarly performance of SVM for the above mentioned features given in Table 3. Classification accuracy for the three rotor speeds of 20Hz, 30Hz and 40Hz are 93.8%, 86.9% and 94.4% respectively when Shannon entropy is used as feature

vector, while for approximate entropy the accuracies are 96.3%, 89.4% and 90.6% respectively. Here, also slightly lesser classification performance is observed when spectral entropy is used as feature vector. The classification performance in terms of percentage accuracy for rotor speed of 20Hz, 30Hz and 40Hz are 86.1, 87.5 and 91.3 respectively for spectral entropy. The number of support vectors (SV) obtained after the optimization is also given for each case in Table 3.

Table2: Classification Accuracy of ANN at different speeds

Feature	Shannon entropy		Approx. entropy		Spectral entropy	
Rotor Speed	Accuracy (%)	Epoch	Accuracy (%)	Epoch	Accuracy (%)	Epoch
20Hz	97.5	21	96.9	32	86.3	33
30Hz	86.9	41	85.6	112	73.3	42
40Hz	93.8	32	96.3	40	91.3	25

Table 3: Classification Accuracy of SVM at different speeds

Feature	Shannon entropy		Approx. entropy		Spectral entropy	
Rotor Speed	Accuracy (%)	No. SV	Accuracy (%)	No. SV	Accuracy (%)	No. SV
20Hz	93.8	88	96.3	49	86.3	103
30Hz	86.9	75	89.4	69	87.5	95
40Hz	94.4	64	90.6	74	91.3	70

Confusion matrices obtained by using IMF entropy feature for different fault conditions are shown in Table 4 and 5 for SVM and ANN respectively when the rotor speed is 20Hz. Total 160 numbers of instances of bearing fault are obtained in which BF, HB, IR and OR consist of 40 cases each. It may be inferred from Table 5 that SVM correctly predicted 36, 40, 40 and 34 instances of BF, HB, IR and OR respectively, while ANN has correctly predicted 40, 40, 38, 38 instances respectively. The number of false predictions can be also observed from Table 4 and 5.

Table 4: Confusion Matrix for SVM with Shannon entropy at 20Hz

Predicted Class	BF	HB	IR	OR
Observed Class				
BF	36	3	0	1
HB	0	40	0	0
IR	0	0	40	0
OR	6	0	0	34

Table 5: Confusion Matrix for ANN with Shannon entropy at 20Hz

Predicted Class	BF	HB	IR	OR
Observed Class				
BF	40	0	0	0
HB	0	40	0	0
IR	0	0	38	2
OR	0	0	2	38

The confusion matrices for approximate entropy as classification feature for SVM and ANN are also given in Table 6 and 7 respectively for rotor speed of 20Hz.

It may be observed from Table 6 that the SVM correctly predicts 37, 40, 40, and 37 cases of BF, HB, IR and OR respectively out of 40 cases each. Similarly, ANN correctly predicts 40, 40, 37 and 38 instances of above fault conditions respectively.

Table 6: Confusion Matrix for SVM with ApEnat 20Hz

Predicted Class	BF	HB	IR	OR
Observed Class				
BF	37	0	0	3
HB	0	40	0	0
IR	0	0	40	0
OR	3	0	0	37

Table 7: Confusion Matrix for ANN with ApEn at 20Hz

Predicted Class	BF	HB	IR	OR
Observed Class				
BF	40	0	0	0
HB	0	40	0	0
IR	0	0	37	3
OR	0	0	2	38

CONCLUSION

The present work deals with fault diagnosis of rolling element bearing based on SVM and ANN. EMD method is applied to extract the so-called Intrinsic Mode Function (IMF) from acceleration signal under faulty and healthy condition. In the work two new fault features are proposed based on complexity measure of the system signal- approximate entropy and spectral entropy. Shannon entropy as a measure of information contain is also extracted. Shannon entropy and approximate entropy features from the IMF signals are shown to be very efficient and accurate for bearing fault diagnosis.

ACKNOWLEDGEMENT

This work is supported by research grants from Department of Science & Technology, New Delhi, India through SERC research scheme (Grant No. SR/FTP/ETA-0040/2011).

REFERENCES

1. B. Samanta and K. R. Al-Balushi, Artificial neural network based fault diagnostics of rolling element bearings Using time-domain features, Mechanical Systems and Signal Processing 17(2)(2003) 317–328.
2. M. Subrahmanyam and C. Sujatha, Using neural networks for the diagnosis of localized defects in ball bearings, Tribology International 30(10) (1997) 739–752.
3. Yaguo Lei, Zhengjia He and Yanyang Zi, A new approach to intelligent fault diagnosis of rotating machinery, Expert Systems with Applications 35 (2008) 1593–1600.
4. J. Altmann and J. Mathew, Multiple band-pass autoregressive demodulation for rolling-element bearing fault diagnosis, Mechanical Systems and Signal Processing 15(5)(2001) 963–977.
5. Hui Li, Yuping Zhang, Haiqi Zheng, Bearing fault detection and diagnosis based on order tracking and Teager-Huang transform, Journal of Mechanical Science and Technology 24(3) (2010) 811–822.
6. Yang Yu, YuDejie, Cheng Junsheng, A roller bearing fault diagnosis method based on EMD energy entropy and ANN, Journal of Sound and Vibration 294 (2006) 269–277.
7. Hongbo Dong, Keyu Qi, Xuefeng Chen, Yanyang Zi, Zhengjia He and Bing Li, Sifting process of EMD and its application in rolling element bearing fault diagnosis, Journal of Mechanical Science and Technology 23 (2009) 2000–2007.
8. R.B.W. Heng and M.J.M. Nor, Statistical analysis of sound and vibration signals for monitoring rolling element bearing condition, Applied Acoustics, 53(1-3) (1998) 211–226.
9. V.K. Rai and A.R. Mohanty, Bearing fault diagnosis using FFT of intrinsic mode functions in Hilbert–Huang transform, Mechanical Systems and Signal Processing 21 (2007) 2607–2615.
10. J.I. Taylor, Identification of Bearing Defects by Spectral Analysis, Trans ASME, J Mech Design 102 (1980) 199–204.
11. D.P. Jena, S.N. Panigrahi, Bearing and gear fault diagnosis using adaptive

- wavelet transform of vibration signals, *Procedia Engineering* 50 (2012), 265-274.
12. P.K. Kankar, Satish C. Sharma, S.P. Harsha, Rolling element bearing fault diagnosis using wavelet transform, *Neurocomputing* 74 (2011) 1638-1645.
 13. P.K. Kankar, Satish C. Sharma, S.P. Harsha, Fault diagnosis of rolling element bearing using cyclic autocorrelation and wavelet transform, *Neurocomputing* 110 (2013) 9-17.
 14. Xiaofeng Liu, Lin Bo, Xingxi He, Martin Veidt, Application of correlation matching for automatic bearing fault diagnosis, *Journal of Sound and Vibration* 331 (2012) 5838-5852.
 15. Xinsheng Lou, Kenneth A. Loparo, Bearing fault diagnosis based on wavelet transform and fuzzy inference, *Mechanical Systems and Signal Processing* 18 (2004) 1077-1095.
 16. P.D. Mcfadden and M.M. Toozhy, Application of Synchronous Averaging To Vibration Monitoring of Rolling Element Bearings, *Mechanical Systems and Signal Processing* 14(6)(2000) 891-906.
 17. Guofeng Wang, Chang Liu and Yinhu Cui, Clustering diagnosis of rolling element bearing fault based on integrated Autoregressive/ Autoregressive Conditional Heteroscedasticity model, *Journal of Sound and Vibration* 331 (2012) 4379-4387.
 18. Long Zhang, Guoliang Xiong, Hesheng Liu, Huijun Zou and Weizhong Guo, Bearing fault diagnosis using multi-scale entropy and adaptive neuro-fuzzy inference, *Expert Systems with Applications* 37 (2010) 6077-6085.
 19. V. Purushotham S. Narayanan and Suryanarayana A.N. Prasad, Multi-fault diagnosis of rolling bearing elements using wavelet analysis and hidden Markov model based fault recognition, *NDT&E International* 38 (2005) 654-664.
 20. N. Sawalhi, R.B. Randall, H. Endo, The enhancement of fault detection and diagnosis in rolling element bearings using minimum entropy deconvolution combined with spectral kurtosis, *Mechanical Systems and Signal Processing* 21 (2007) 2616-2633.
 21. N. E. Huang, Z. Shen, S.R. Long, M. C. Wu, H.H. Shih, Q. Zheng, N.C. Yen, C.C. Tung, H.H. Liu, The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis, *Proceedings of the Royal Society London* 454 (1998) 903-995.
 22. N. E. Huang and Z. Wu, A review on Hilbert-Huang transform: Method and its applications to geophysical studies, *Reviews of Geophysics* 46(2008) 1-23.
 23. S.M. Pincus, Approximate entropy as a measure of system complexity, *Proceedings of the National Academy of Sciences of the United States of America* 88(6) (1991) 2297-301.