Selection of Gear Materials Using MCDM-TOPSIS Approach

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ABSTRACT
Materials selection are a key issue in the optimal design of the industrial product. Material selection is a difficult and subtle task because of an immense number of different available materials. Increasing demand for gears in the different field of applications has forced manufacturers to design and develop gears which compete in the global market and satisfy the user requirements as well as environmental conditions. Various attributes of gear materials which are generally considered by a consumer are density, tensile strength, shear modulus, yield strength, brinell hardness and cost so as to provide consumers desired value and satisfaction level. Currently, numerous gear materials are available in the market having their own merits and demerits, appropriate methodology for selection is essential. An MCDM-TOPSIS Approach has been used in the paper to select the appropriate material for gear manufacturing considering several criteria.

Keywords: gear, gear material, multicriteria decision making, relative importance matrix, TOPSIS

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INTRODUCTION

Today we see development in every aspect of society. The important task in the early stage gear materials design is to determine gear material configuration. The configuration of gear material includes the decision of material type, the selection of component and the choice of control strategies. Almost every gear design project, there is more than one alternative configuration that would meet the design requirement. Also, among the globe because of the rapid increase in the number of gear and gear manufacturers, the selection of gear material for particular application and selection of best alternative gear configuration to meet the design requirements has become a difficult task for user as well as designer of gear materials. The performance is key criteria for the selection of gear material but also there are many other parameters such as reliability, quality, availability, cost etc. are important during the selection of gear material. Only few literatures with few considering parameters available for the selection of gear material so there is a need of particular approach for selection of the optimal gear material considering the given alternatives [1–4].

MCDM-TOPSIS APPROACH

Decision making is an issue which is faced by customers, manufacturers and organisation managers for achieving complete customer satisfaction on the basis of cost and reliability. One state of the art technique which is used to make decisions is the multi criteria decision making or MCDM. It is the branch of Operations Research which is used for making decisions based on multiple objectives. These models can handle quantitative as well as qualitative criteria.

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a widely used technique for Multi Criteria Decision Making (MCDM). It has been proved to be an efficient technique for evaluating, assessing and ranking alternations across all industries [5–7]. The basic principle is that the chosen alternative must have the shortest distance from the ideal solution and the farthest distance from the negative ideal solution [8]. Bhagale and Agrawal [9] have used MCDM-TOPSIS approach for the selection of robot including criteria as complexities, features, reliability, availability etc. Adhikari and Roy [10] used MCDM-TOPSIS for selection of hydro turbine blade material. Brar [11] used TOPSIS approach for selection of suitable air conditioning system. A survey of TOPSIS applications has been very systematically done by Behzadian et al. [12]. The procedure is carried out in the following steps as given by Brar [11] and Adhikari and Roy [10].

Evaluation Procedure

A shortlist of gear material alternatives [13] formed as a result of elimination search has to be further filtered to find out the best solution out of all. Hence these accessible substitutes are ranked in order of preference to select an optimal one.

Decision Matrix

Firstly, all of the information available from the mini database about these satisfying solutions is represented in the matrix form. Such a matrix is termed as a decision matrix, 'D'. Each row of the matrix is allocated to one alternative gear material and each column to one attribute. Therefore, an element dij of the decision matrix, 'D' represents the value of j^{th} attribute in non-normalized form/units, corresponding to i^{th} alternative. Thus, if there are 'm' short-listed substitutes with 'n' pertinent attributes, the decision matrix is an m x n matrix.

Normalized Matrix

As the elements in each column of matrix, 'D' have different units and scales, it is necessary to normalize their values. Thus, normalized matrix, 'N' is constructed to have the dimensionless magnitudes of all the attributes of gear materials on common scale of 0 to 1, which allows the comparison across the attributes. Each element n_{ij} of the normalized matrix, 'N' can be calculated as

$$nij = \frac{dij}{\sqrt[2]{\sum_{i=1}^{m} dij^2}} \qquad (1)$$

where d_{ij} is an element of the decision matrix, 'D'.

Relative Importance Matrix

In this step, the relative importance matrix 'R' of size n x n is formed to incorporate the relative importance of the attributes over others for a given application. An element r_{ij} of matrix 'R' represents the relative importance of the i^{th} attribute over the j^{th} attribute and is defined as

$$r_{ij} = \frac{Importance\ of\ ith\ attribute}{importance\ of\ jth\ attribute} \qquad (2)$$

The relative importance of one attribute with respect to another for a given application can be obtained from the user or the group of experts specialized in a particular application. The information about the pair-wise comparison of attributes for a particular application is stored in this relative importance matrix 'R', with all its diagonal elements as unity.

Eigen Value Formulation and Weight Matrix

Due to human inconsistencies, the information stored in the 'R' matrix on a pair-wise basis cannot be used directly. It must be modified into a form that gives the relative weights of all attributes taken together so that the sum of all the weight is equal to unity. Thus, eigen value formulation is used to find weight vector matrix, 'W' and is expressed as

$$RW = \lambda W \qquad (3)$$

where, $W = \{w_1, w_2, w_3 \ldots w_n\}^T$, and λ is the eigen values.

Eq. (3) can be expressed

$$(R-\lambda I)W=0 \quad (4)$$

To avoid the trivial solution, we have

$$\text{Det}\,(R-\lambda I) = 0 \quad (5)$$

The solution of Eq. (5) gives the set of 'n' eigen values ($\lambda_1, \lambda_2, \ldots, \lambda_n$). The solution of Eq. (4) for the maximum eigen value 'λ_{max}' gives the weight matrix, 'W' and the expression is given as

$$(R-\lambda I)W = 0 \quad (6)$$

Weighted Normalized Decision Matrix

In this step the weighted normalized decision matrix, 'V' is obtained by incorporating the information stored in the weight matrix, 'W' into the normalized matrix, 'N'. A true comparable value of each attribute is given by this weighted normalized matrix and is defined as

$$V = [v_{ij}], \text{ where } v_{ij} = w_j \times n_{ij}, \quad (7)$$

where $i = 1,2,\ldots,m$; $j = 1,2,\ldots,n$

Hypothetical Best and Worst Solution

The hypothetical best solution (HBS) and hypothetical worst solution (HWS) are determined by choosing the maximum and minimum values of attributes from 'V' matrix as

HBS = A* = v_{ij} *max*, for benefit attributes (Larger the better type), or
= v_{ij} *min*, for cost attributes
(Smaller the better type), and (8)

HWS = A¯ = v_{ij} *min*, for benefit attributes (Larger the better type), or
= v_{ij} *max*, for cost attributes
(Smaller the better type) (9)

Where $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$.
Hence,

$A^* = (V^*_1, V^*_2, \ldots, V^*_n)$

$A^- = (V^-_1, V^-_2, \ldots, V^-_n)$

Determination of Separation Measures

The TOPSIS procedure is based on the concept that the chosen option should be nearest to the HBS and farthest from the HWS. The separation measure of top ranked gear material ensures that it is closest to the HBS (best possible gear material) and farthest from the HWS (worst possible gear material). If Si* and Si¯ are separation measures from HBS and HWS, respectively. Then, the separation of each alternative from the HBS is given by

$$Si^* = [\textstyle\sum_{j=1}^{n}(v_{ij} - v_j^*)2]\ ½\ (i = 1, 2, 3\ldots, m) \quad (10)$$

And separation measure from HWS is given by

$$Si^- = [\textstyle\sum_{j=1}^{n}(v_{ij} - v_j^-)2]\ ½\ (i = 1, 2, 3\ldots, m) \quad (11)$$

Determination of Suitability Index

The suitability index, 'C*' is a measure of the suitability of the gear material for the chosen application on the basis of attributes considered. It is defined as the relative closeness to the HBS, and is expressed as

$$C^* = Si^-/(Si^* + Si^-),\ i = 1, 2\ldots\ldots m \quad (12)$$

An air conditioning system with largest C* is preferable.

Establishing an Order of Preference

The gear material with highest value of C* will be given highest rank, and so on. In this way the preference order for the available alternative gear materials is obtained by arranging them in decreasing order of their corresponding C* values.

SELECTION OF GEAR MATERIAL

Different alternatives available are Cast iron, Al alloy, Cu alloy, Ni alloy, Zn alloy, and Nylon 6. TOPSIS technique is used for selection of gear material based upon attributes like density, tensile strength, shear modulus, yield strength, brinell hardness and cost. These attributes for candidate gear material are given in following Table 1 [14–17]. The selection procedure is carried out as per the steps of MCDM-TOPSIS method.

Step I

Table 1. *Attributes for candidate material of gear manufacturing.*

Attributes / Alternatives	Density (kg/m^3)	Tensile strength (MPa)	Shear modulus (GPa)	Yield strength (MPa)	Brinell hardness	Cost/kg (Rupees)
Cast iron	7250	884	47.5	757.7	463.5	45
Al alloy	2700	304	25	265	80	100
Cu alloy	8930	325	28	265	44	300
Ni alloy	8830	772.5	25	585	700	883
Zn alloy	5000	327.5	25	265	82	450
Nylon 6	9380	64.5	15	216	183	30

Step II

Decision matrix

$$D = \begin{pmatrix} 0.1722 & 0.330 & 0.285 & 0.336 & 0.2985 & 45 \\ 0.0641 & 0.1135 & 0.1510 & 0.1175 & 0.0515 & 100 \\ 0.2121 & 0.121 & 0.1691 & 0.1175 & 0.2834 & 300 \\ 0.2097 & 0.2885 & 0.1510 & 0.2595 & 0.450 & 883 \\ 0.118 & 0.1223 & 0.1510 & 0.1175 & 0.528 & 450 \\ 0.222 & 0.024 & 0.0906 & 0.0514 & 0.117 & 30 \end{pmatrix}$$

Step III

Normalizing matrix

$$N= \begin{pmatrix} 0.3995 & 0.680 & 0.660 & 0.709 & 0.4768 & 0.0432 \\ 0.1487 & 0.234 & 0.3497 & 0.2480 & 0.0822 & 0.0960 \\ 0.49211 & 0.2495 & 0.3917 & 0.2480 & 0.4527 & 0.288 \\ 0.4865 & 0.5949 & 0.3497 & 0.5479 & 0.7201 & 0.8477 \\ 0.273 & 0.252 & 0.349 & 0.248 & 0.084 & 0.0432 \\ 0.515 & 0.0494 & 0.2099 & 0.1085 & 0.188 & 0.0288 \end{pmatrix}$$

Step IV

Relative importance matrix

$$R= \begin{pmatrix} 1 & 1/3 & 1/3 & 1/4 & 1 & 1 \\ 3 & 1 & 2 & 2 & 1 & 3 \\ 3 & 1/2 & 1 & 1 & 1 & 1/3 \\ 4 & 1/2 & 1 & 1 & 1/3 & 1 \\ 1 & 1 & 1 & 3 & 1 & 1 \\ 1 & 1/3 & 3 & 1 & 1 & 1 \end{pmatrix}$$

Weight for each alternative can be calculated by calculated the square of eigen vectors corresponding to maximum eigen value (λ max=6.7701) of matrix R, which as equal to

(0.0453, 0.3831, 0.1052, 0.121, 0.2004, 0.1439)

Step V

Weight normalized matrix

$$V= \begin{pmatrix} 0.0180 & 0.260 & 0.069 & 0.149 & 0.095 & 0.0062 \\ 0.0067 & 0.089 & 0.036 & 0.030 & 0.0164 & 0.0138 \\ 0.022 & 0.095 & 0.036 & 0.030 & 0.090 & 0.0414 \\ 0.021 & 0.227 & 0.036 & 0.066 & 0.144 & 0.1219 \\ 0.012 & 0.096 & 0.0367 & 0.0300 & 0.016 & 0.0621 \\ 0.023 & 0.0189 & 0.022 & 0.013 & 0.037 & 0.0041 \end{pmatrix}$$

Step VI

Hypothetical best and worst solution

V^*= (0.0067, 0.0189, 0.022, 0.013, 0.026, 0.0041)

V^-= (0.023, 0.260, 0.069, 0.149, 0.144, 0.1219)

Step VII

Determination of separation Measure

$S_1^*=0.288$	$S_1^-=0.1185$
$S_2^*=0.0747$	$S_2^-=0.269$
$S_3^*=0.109$	$S_3^-=0.0514$
$S_4^*=0.272$	$S_4^-=0.0952$
$S_5^*=0.1004$	$S_5^-=0.248$
$S_6^*=0.019$	$S_6^-=0.322$

Step VIII
Determination of suitability index
C1*=0.2910
C2*=0.7826
C3*=0.3204
C4*=0.2592
C5*=0.7122
C6*=0.9443

Step IX
Ranking of alternatives

Alternatives	Ranking
Nylon 6	1
Al alloy	2
Zn alloy	3
Cu alloy	4
Cast iron	5
Ni alloy	6

RESULT AND DISCUSSION
Among the available alternatives considering various criteria, Nylon 6 material has been found to be a most preferred choice among the six material alternatives. The TOPSIS method, at the first stage, consists of the composition of the decision matrix with the values of attributes (criteria) like density, tensile strength, shear modulus, yield strength, brinell hardness and cost. Based on the above matrix, the normalized decision matrix is constructed. Weighted normalized decision matrix has been obtained by using the normalized decision matrix and weights assigned to criteria. The used methodology will be very useful to find candidate material for various applications and also for other field of applications.

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