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Review IJMD

Robotic Path: Analysis of Algebraic Topological Properties of Stream Lines Due to Source-Sink, Dipole and Vortex

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Abstract

The importance of robotic path planning cannot be overstated. Robotic path planning plays a crucial role in ensuring efficient and safe navigation for robots. It enables robots two autonomously determine the most optimal path to reach their destination while avoiding of obstacles and potential collisions. By carefully analyzing the environment and considering various factors such as distance, speed, and obstacles, robotic path planning enhances the overall performance and productivity of robots. Moreover, it minimizes the risk of accidents and damage to both the robots and their surroundings. In industries such as manufacturing logistics and health care where robots are extensively used, effective path planning is essential for seamless operations and improved efficiency. Therefore, investing in advance robot planning terms and technology is crucial to unlock the full potential of robotics and automation Due to ubiquity and exquisite properties: source, sink, dipole and vortex are central concepts in fluid mechanics and hence the fluid flows are outright concurrent with it. Source and sink in tandem manifest the crucial properties. Thus, the discussion pervades in almost every branch of science, tempted the researcher and compel to work. One such attempt is made in this paper, to study the algebraic topological properties like homotopy, null homotopy and fundamental group of stream lines of flow due source-sink, dipole and vortex which can render analytic simulation of natural phenomenon as well as robotic path planning.

Keywords: Robotic Path, Source-Sink, Homotopy, Fundamental group, Vertex, Dipole.

INTRODUCTION

The existence of source and sink is highly interwoven with the existence of universe. As there are lot of phenomena in nature consisting source and sink. The cosmologist Prof. Mohsen Sharifpur in his “Source and Sink theory” [1] reveals that the energy which created the universe should be a pulse of a point source (the Big Bang) and posits that there should be sink towards which the radiations marching (following the laws of fluid dynamics and energy is conserved) due to its gravity. The zoological flows such as flow of urine in urethras, blood flow in veins or arteries and air flow in the bronchial airways [2] are such flows. Plant growth and development generally depends on photosynthetic resources. Photosynthesizing tissues transporting carbon may be defined as source and the sink is the one that imports carbon. The strength of source/sink is the ability to mobilize the photo-assimilates. Thus, the parts of plants are also allocated as source and sink depending upon the translocation of assimilates through phloem [3, 4]. The analogue of cyclonic or anti-cyclonic gyre in laboratory is studied by Han-Hsiung Kuo et al. [5]

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also tropical cyclones are modeled as source sink and vortex by Mayumi Yoshimoto et al. [6]. However, tiny nano-particle are not precluded from this discussion [7].

Being a dynamic outcome of artificial intelligence, robotic path planning is among the important field of research in Mechatronics which depends on functions of two variables defining velocity. The velocity potential is used in fluid mechanics to describe the flow of an incompressible fluid. By analyzing the velocity potential, one can determine the velocity components of the fluid flow and gain insights into the behavior of the fluid. As the path of robot is analogous to stream lines of flowing particle, its applications are portraited for robot path planning in [8–11]. In this paper complex velocity potential of some flows due to source, sink, vertex and doublet [12] is obtained and homotopy [13] of the stream lines is established. Isomorphism of Fundamental group [13] of stream lines of flow around circular cylinder is studied. Stream lines of flows obtained from complex velocity potential used as path of robot.

The fluid model for robotic path planning is an innovative and unique concept that provides a new perspective on robot navigation and interaction. Traditionally, robots have been viewed as inflexible machines that follow preset paths. However, the fluid model challenges this by considering robots as adaptable entities that can respond to their surroundings in a fluid-like way. In this model, robot movement is compared to fluid flow, where the path emerges from the interaction between the robot and environment instead of being fixed beforehand. Just as a fluid changes shape based on external forces, robots can dynamically adjust their route based on environmental constraints and forces. The fluid model has many applications across fields. In manufacturing, it allows robots to move fluidly around obstacles, enhancing productivity. In healthcare, it enables robots to nimbly traverse complex settings like hospitals to assist with patient care and logistics. Furthermore, the model can be applied to swarm robotics, where multiple robots coordinate. Viewing the swarm's collective behavior as a fluid provides insights into optimizing overall movement and coordination, improving efficiency and scalability.

MAIN THEOREM

Theorem

Stream lines of the flow due to source and sink of same strength are homotopic

Proof

The complex potential of flow from source z_1 to sink z_2 of same strength m,

$$\Omega(\mathrm{z}) = m \log(\mathrm{z} - z_1) - m \log(\mathrm{z} - z_2)$$
$$= m \log\left(\frac{z-z_1}{z-z_2}\right)$$

∴ The stream lines are

$$\Psi(x, y) = Im\, \Omega(\mathrm{z}) = c, \text{constant}$$
$$m \arg\left(\frac{z-z_1}{z-z_2}\right) = c$$
$$\arg\left(\frac{z-z_1}{z-z_2}\right) = \mu \ (1)$$

where $\dfrac{c}{m} = \mu = \text{constant}$

Equation (1) is the locus of all points z that makes an angle μ when subtended by the chord z_1 z_2 which is geometrically the arc of the circle from z_1 to z_2.

To find the radius r and center z_0 of the circle. Since,

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$$|z_0 - z_1| = |z_0 - z_2|$$

$$z_0 - z_1 = (z_0 - z_2)e^{2i\mu}$$

$$z_0 = \frac{z_1 - z_2 e^{2i\mu}}{1 - e^{2i\mu}}$$

$$r = |z_0 - z_1| = |z_0 - z_2|$$

Take the two stream lines $f(s)$ and $g(s)$.

$$H(s,t) = t\, g(s) + (1-t) f(s)$$

is the required homotopy.

Example

If $z_1 = 1$ and $z_2 = -1$ are the source and the sink of same strength m, then the stream lines are homotopic.

Solution: The complex potential, when the source and sink of strength m are placed at $\boldsymbol{z_1 = 1}$ and $\boldsymbol{z_2 = -1}$ respectively, is given by the equation

$$\Omega(z) = m \log(z-1) - m \log(z+1)$$

$$= m \log\ [(x-1) + iy] - m \log[(x+1) + iy]$$

Therefore, the stream lines are

$$m\left[\tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right)\right] = c$$

$$\tan^{-1}\left[\frac{(x+1)\, y - y(x-1)}{(x^2-1) + y^2}\right] = \frac{c}{m}$$

$$\frac{2\, y}{(x^2-1) + y^2} = \tan \mu$$

where $\mu = \dfrac{c}{m}$

$$x^2 - 1 + y^2 = 2y\ \cot \mu$$

$$x^2 + (y-k)^2 = 1 + k^2$$

where $k = \cot \mu$. The stream lines in complex form are

$$z = ik + \sqrt{1 + k^2}\, e^{i\theta},\ 0 \le \theta \le 2\pi$$

Now consider two stream lines $f(s)$ and $g(s)$ for k = 0 and k = 1 respectively.

$$f(s) = e^{is\pi},\ g(s) = 1 + \sqrt{2} e^{i\,(\frac{7\pi}{4} - \frac{s\pi}{2})}$$

Thus, the required homotopy is

$$H(s,t) = t f(s) + (1-t) g(s)$$

Above homotopy is pictorially shown in (Ref. Figure 1)

Theorem

If z_1 is a dipole of strength μ, then the stream lines of flow due to dipole are null homotopic.

Proof

The complex potential for the flow of fluid due dipole at $z_1 = x_1 + iy_1$ of strength μ is

$$\Omega(z) = \frac{\mu\, e^{i\alpha}}{z - z_1}$$

$$= \frac{\mu\,[\cos\alpha + i\,\sin\alpha\,]}{(x-x_1) + i(y-y_1)}$$

$$= \frac{\mu\,(\cos\alpha + i\sin\alpha)\,[\,(x-x_1) - i(y-y_1)}{(x-x_1)^2 + (y-y_1)^2}$$

$$\Omega(\mathrm{z}) = \frac{\mu\{[(x-x_1)\cos\alpha + (y-y_1)\sin\alpha] + i\,[(x-x_1)\sin\alpha - (y-y_1)\cos\alpha]\}}{(x-x_1)^2 + (y-y_1)^2}$$

The stream lines are given by

$$\frac{\mu[(x-x_1)\sin\alpha - (y-y_1)\cos\alpha]}{(x-x_1)^2 + (y-y_1)^2} = c$$

where c is a constant.

$$(x - x_1)^2 + (y - y_1)^2 = 2\,k\,[(x - x_1)\sin\alpha - (y - y_1)\cos\alpha\,]$$

where $2k = \frac{\mu}{c}$.

$$[x - (x_1 + k\,\sin\alpha)]^2 + [y - (y_1 - k\,\cos\alpha)]^2 = k^2$$

which are circles of radius k and centre $(x_1 + k\sin\alpha,\; y_1 - k\cos\alpha)$

∴ Centre, $z_0 = (x_1 + k\sin\alpha) + i(y_1 - k\,\cos\alpha)$

$$z_0 = z_1 - i\,k\,e^{i\,\alpha}$$

Consider two stream lines for $k = 0$ and $k = 1$.

$$f(s) = z_1,\, g(s) = z_0 + e^{2\pi i s},\; 0 \le s \le 1.$$

$$H(s,t) = tf(s) + (1-t)g(s)$$

is the required null-homotopy.

Example

If a doublet (dipole) of strength m per unit length is placed at origin and its axis being along x-axis, then the stream lines are null-homotopic.

Solution: Let O be the doublet of strength m per unit length and its axis is along x-axis

$$\Omega(\mathrm{z}) = \frac{\mu}{z} = \frac{\mu\,(x - i\,y)}{x^2 + y^2}$$

The stream lines are

$$\frac{-\,\mu\,y}{x^2 + y^2} = c$$

$$x^2 + y^2 - 2\,k\,y = 0$$

$$x^2 + (y-k)^2 = k^2$$

$$z = i\,k + k\,e^{i\theta}, \quad 0 \le \theta \le 2\pi$$

Consider two stream lines for $k = 0$ and $k = 1$.

$$f(s) = 0,\, g(s) = i + e^{2\pi i s},\; 0 \le s \le 1.$$

The null-homotopy between $f(s)$ and $g(s)$ is

$$H(s,t) = tf(s) + (1-t)g(s)$$

(Ref. Figure 2).

Theorem

If the vortex of strength m is kept at z_1, then the stream lines of flow are null-homotopic.

Proof

Let $z_1 = x_1 + iy_1$ be the vortex, the complex velocity potential is given by

$$\Omega\,(z) = -\frac{i\Gamma}{2\pi}\log(z - z_1)$$

If $|z - z_1| = r$ then stream lines are

$$\frac{\Gamma}{2\,\pi}\log r = c$$

$$\log r = \frac{2\,\pi c}{\Gamma}$$

$$r = e^{2\pi c/\Gamma}$$

$$\sqrt{(x - x_1)^2 + (y - y_1)^2} = k,$$

where $k = e^{2\,\pi\,c/\Gamma}$

$$(x - x_1)^2 + (y - y_1)^2 = k^{\,2}$$

The stream lines in complex form are

$$z = z_1 + k\,e^{i\,\theta},\ 0 \le \theta \le 2\pi$$

we consider two stream lines for k = 0 and k = 1.

$$f(s) = z_{1,}\ g(s) = z_1 + e^{2\,\pi\,i\,s},\ 0 \le s \le 1,$$

The straight line homotopy is the required null homotopy.

Theorem

Fundamental group of stream lines of flow around circular cylinder is isomorphic to Z.

Proof

For the sake of convenience take base radius of cylinder as 1 and center at origin of the xy-plane. Obviously, boundary is the first stream line and hence the unit circles. Thus, $\pi_1(S^1)$ = Z ■

To describe the stream line with a form $f(s)$ in the parameter s is not always possible. This situation occurs when the stream line has no defined geometric shape. In such cases, it is feasible to describe the homotopy of streamlines by the concatenation of stream lines.

Theorem

A source of strength m is situated in the middle of two sink each of strength m. Prove that the stream lines are homotopic.

Proof

Without loss of generality, let us suppose that the source of strength m is placed at $O = (0,0)$ in between two sinks, each of strength m which are situated at $z_1 = (-a, 0)$ and $z_2 = (a, 0)$.

The complex potential is

$$\begin{aligned}\Omega(\mathrm{z}) &= 2\ \mathrm{m}\log \mathrm{z} - \mathrm{m}\log(\mathrm{z} - z_1) - \mathrm{m}\log(\mathrm{z} - z_2)\\ &= \mathrm{m}\log z^2 - \mathrm{m}\log(\mathrm{z} + a) - \mathrm{m}\log(\mathrm{z} - a)\\ &= \mathrm{m}\,[\log \mathrm{z}^2 - \log(\mathrm{z}^2 - a^2)]\end{aligned}$$

$$= \text{m} \left[\log\left(x^2 - y^2 + 2ixy\right) - \log\left(x^2 - y^2 - a^2 + 2ixy\right]$$

The stream lines are

$$m\left[\tan^{-1}\left(\frac{2xy}{x^2 - y^2}\right) - \tan^{-1}\left(\frac{2xy}{x^2 - y^2 - a^2}\right)\right] = c$$

$$\tan^{-1}\left(\frac{-2a^2xy}{(x^2 + y^2)^2 - a^2(x^2 - y^2)}\right) = \frac{c}{m}$$

$$\frac{\left(x^2 + y^2\right)^2 - a^2(x^2 - y^2)}{-2a^2xy} = cot\,\frac{c}{m}$$

$$(x^2 + y^2)^2 = a^2(x^2 - y^2 + kxy)$$

where $k = -2\cot\dfrac{c}{m}$.

Clearly stream lines are the loops symmetric about troika x-axis, y-axis and the origin O. So, we prove the homotopy by concatenation of paths. By theorem (2.1) the stream line due to sink z_1 and source O and that of sink z_2 and source O are homotopic, say $f \sim f_1$ and $g \sim g_1$. Thus, $f\bar{g} \sim f_1\bar{g}_1$.

Homotopy of stream lines is shown in figure (Ref. Figure 3) for $a = 1$.

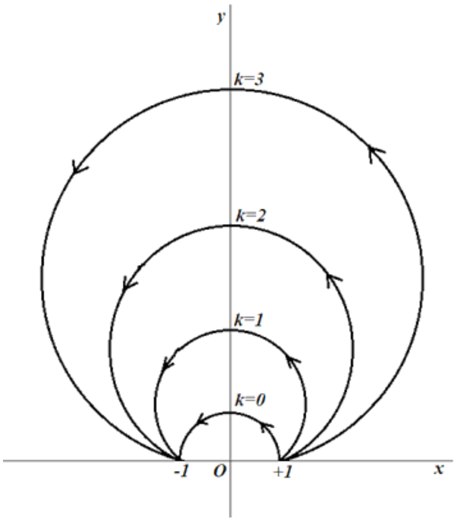

Figure 1. Homotopy of stream lines f(s) and g(s) for k = 0 and k = 1.

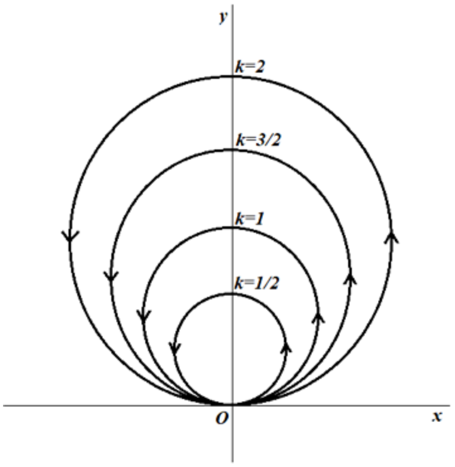

Figure 2. Null-homotopy between f(s) and g(s).

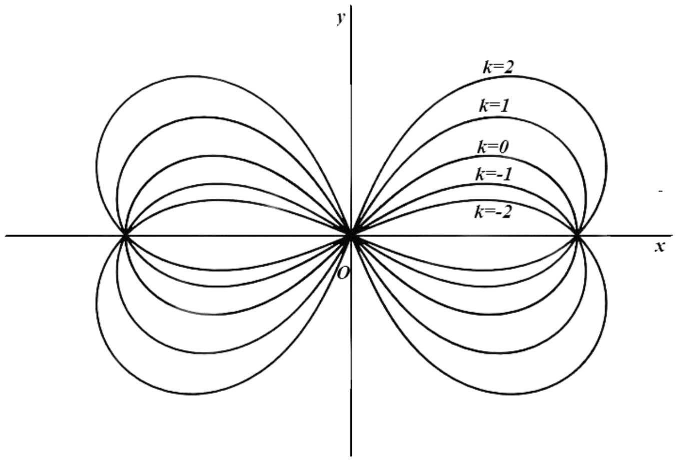

Figure 3. Homotopy of stream lines for a=1.

CONCLUSION

Analogue of stream lines of flow due to source-sink can be used for the robotic path planning in real time (Source and sink are considered as initial position and destination of robot and stream line as path.). Also, realistic human path planning can be made easy through this analogue which in general military march requires in order to reach the goal in minimum available resources. The only limitation is stream lines of flows studied in this paper could be useful for the simulation if there is no obstacle from source to sink, or in the domain of flows of vortex and dipole. If $\pi_1(S^1) = Z$ then there is cylindrical obstacle. If the flows are such that $\pi_1(X) = 0$ (there may be vertex or dipole), then there is no immersed body in the flow, where X the set of stream lines of the flow. Thus, the path can be planned as there is no obstacle. The fluid model for robotic navigation provides a novel and creative viewpoint on how robots move through and engage with their surroundings. By utilizing concepts from fluid mechanics, this framework gives understanding into streamlining robot motions, increasing effectiveness, and boosting adaptability across different uses. This methodology could transform robotics and open up avenues for more sophisticated and flexible robotic frameworks. The fluid perspective on robot pathfinding offers an original and pioneering outlook on robot navigation and interaction. Leveraging fluid dynamics principles furnishes insights on enhancing robot maneuverability, efficiency, and versatility for assorted tasks. This forward-thinking approach may revolutionize robotics and enable more advanced, adaptable robotic systems.

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